Fábián Gábor, ELTE-IK Numerikus Analízis Tanszék, 2014

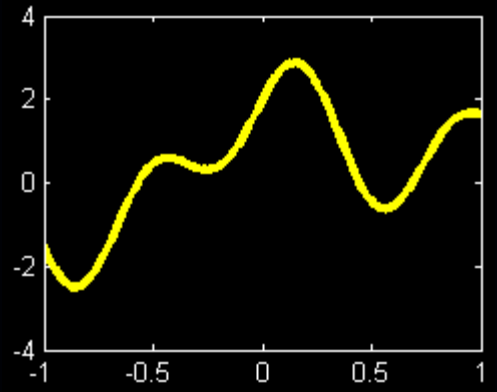
FIZIKAI SZIMULÁCIÓ

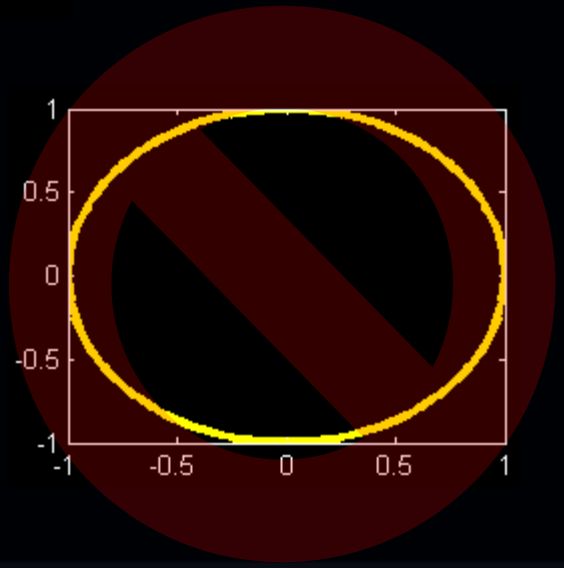
Fizikai Szimuláció

- Fizika
- Függvények
- Analízis
- Deriválté (?!)
- Numerikus Analízis

Függvények

$$x(t)$$

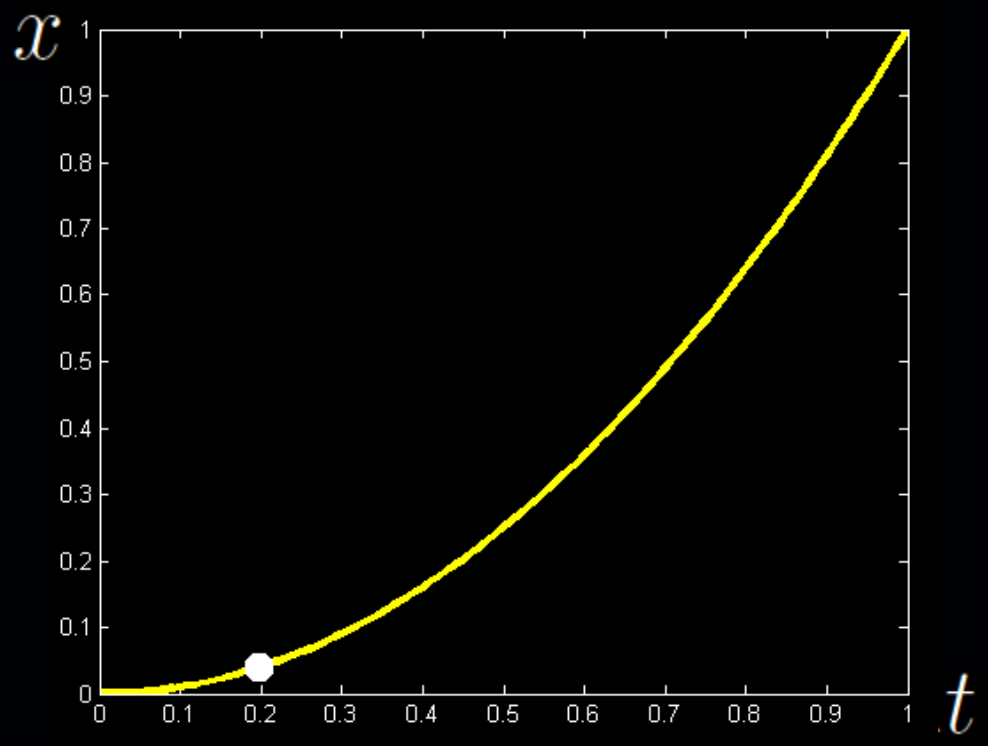




Deriválthányados = Differenciálhányados

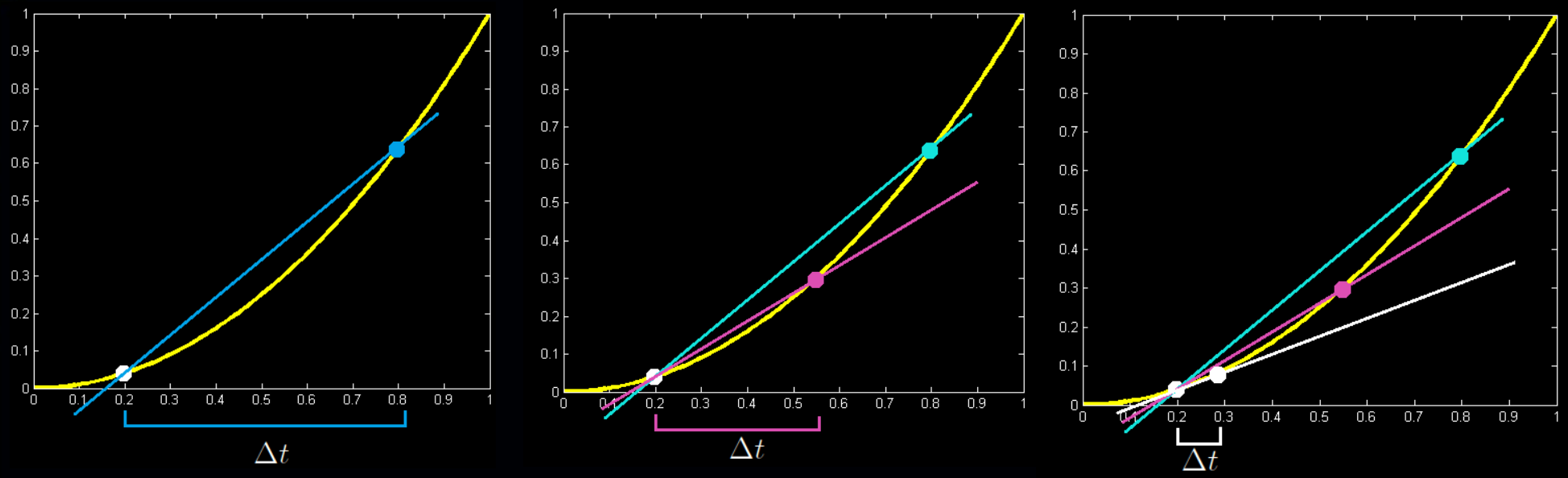
$$\frac{\Delta x}{\Delta t} = \dot{x}$$

$$\Delta t \to 0$$



Deriválť = Differenciálhányados

$$\Delta t \to 0$$



Differenciálegyenletek

$$\dot{x} = f(t, x)$$

Differenciálegyenletek

$$\dot{x} = f(t, x)$$

$$\frac{\Delta x}{\Delta t} = \dot{x}$$

$$\Delta t \to 0$$

Differenciálegyenletek

- Kémiai folyamatok (pl. radioaktív bomlás)
- Biológiai folyamatok (pl. zsákmány-ragadozó modell)
- Pénzügyi folyamatok (pl. tőzsde)
- Dinamika (pl. pontrendszerek, merev testek, folyadékok)
- (Időbeli) változás

Dinamika

$$v = \frac{\Delta x}{\Delta t} = \dot{x}$$

$$a = \frac{\Delta v}{\Delta t} = \dot{v}$$

$$a = \ddot{x}$$

Probléma

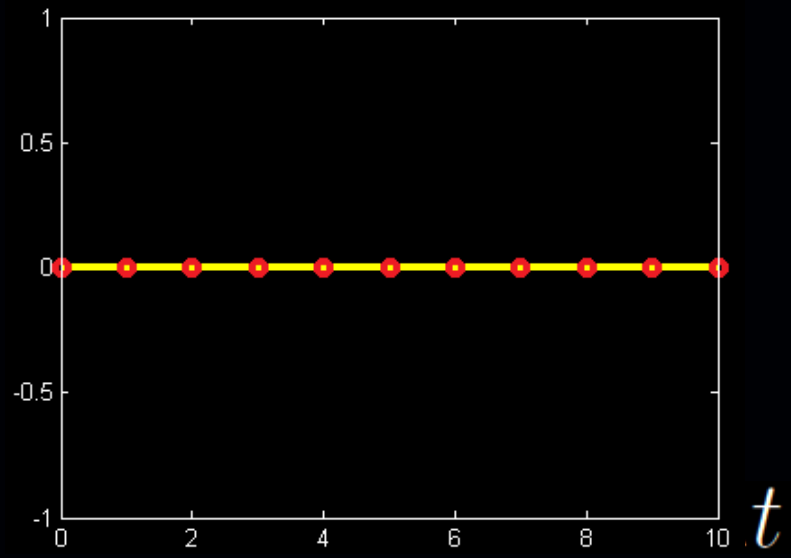
- Nincs megoldás
- Nem ismert a megoldás
- $\Delta t \to 0$

$$FLT_MIN = 1.175494351e - 38$$
$$FLT_DBL = 2.2250738585072014e - 308$$

Diszkretizáció

$$t \in [0, +\infty)$$

$$t_0 = 0$$
$$t_1 = t_0 + h$$
$$t_2 = t_1 + h = t_0 + 2h$$
$$t_3 = t_2 + h = t_1 + 2h = t_0 + 3h$$
$$t_n = t_0 + nh = nh$$



Numerikus közelítés

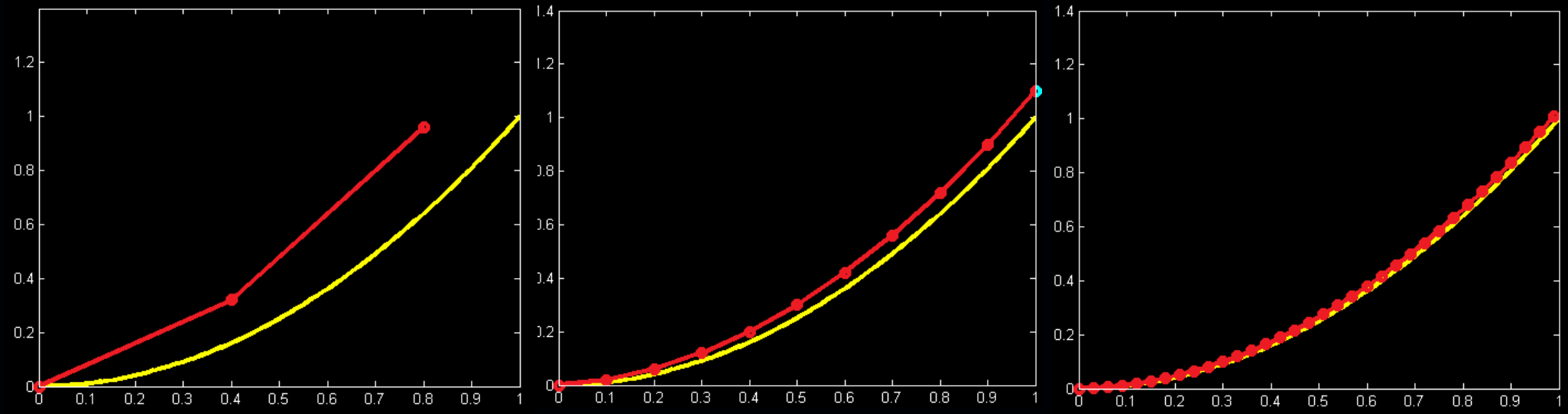
$$t \in [0, +\infty)$$

$$\dot{x} = f(t, x)$$

$$t_0 = 0$$
$$t_n = t_0 + nh = nh$$

$$x_0 = x(0)$$
$$x_{n+1} = x_n + hf(t_n, x_n)$$

Numerikus közelítés



Mozgásegyenletek

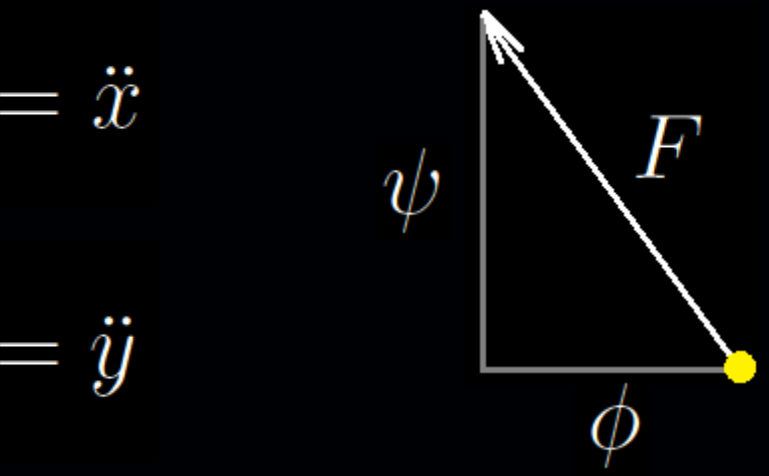
$$F = ma$$

$$\frac{F}{m} = a = \ddot{x}$$

Mozgásegyenletek

$$\frac{\phi}{m} = \ddot{x}$$

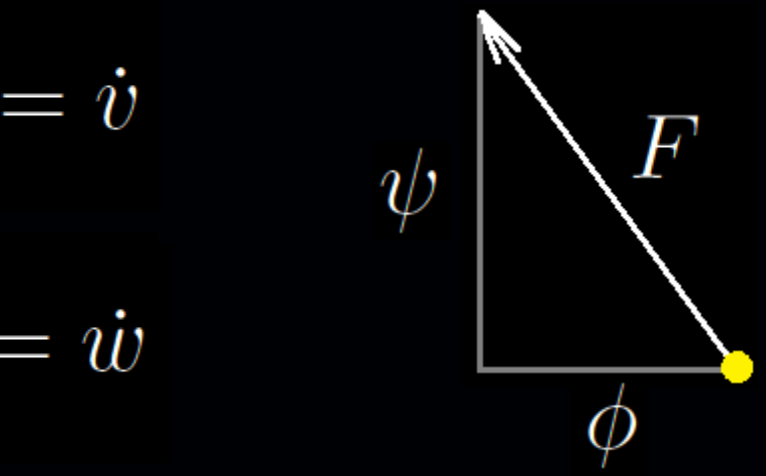
$$\frac{\psi}{m} = \ddot{y}$$



Mozgásegyenletek

$$\frac{\phi}{m} = \dot{v}$$

$$\frac{\psi}{m} = \dot{w}$$

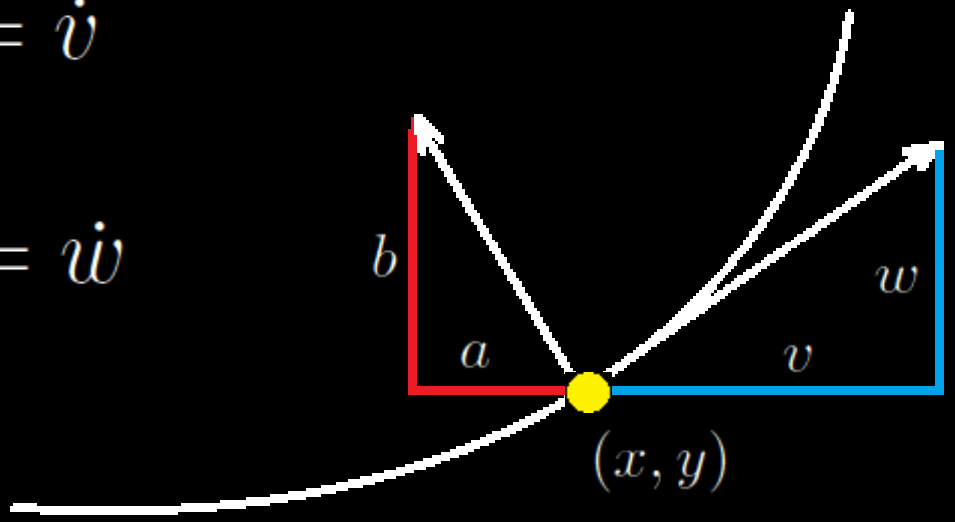


$$\dot{x} = f(t, x) \Rightarrow x_{n+1} = x_n + hf(t_n, x_n)$$

Mozgásegyenletek

$$v = \frac{\Delta x}{\Delta t} = \dot{x} \qquad a = \frac{\Delta v}{\Delta t} = \dot{v}$$

$$w = \frac{\Delta y}{\Delta t} = \dot{y} \qquad b = \frac{\Delta w}{\Delta t} = \dot{w}$$



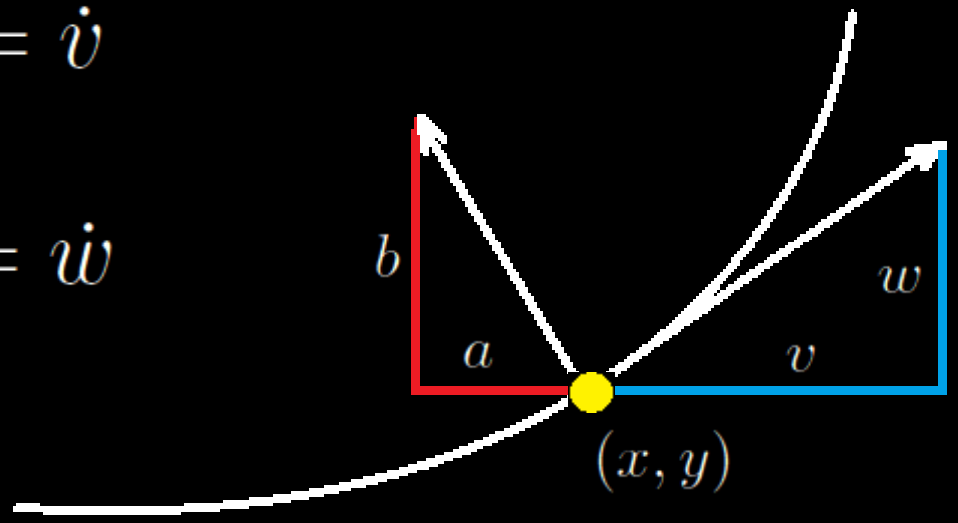
Mozgásegyenletek

$$v = \frac{\Delta x}{\Delta t} = \dot{x}$$

$$w = \frac{\Delta y}{\Delta t} = \dot{y}$$

$$\frac{\phi}{m} = \dot{v}$$

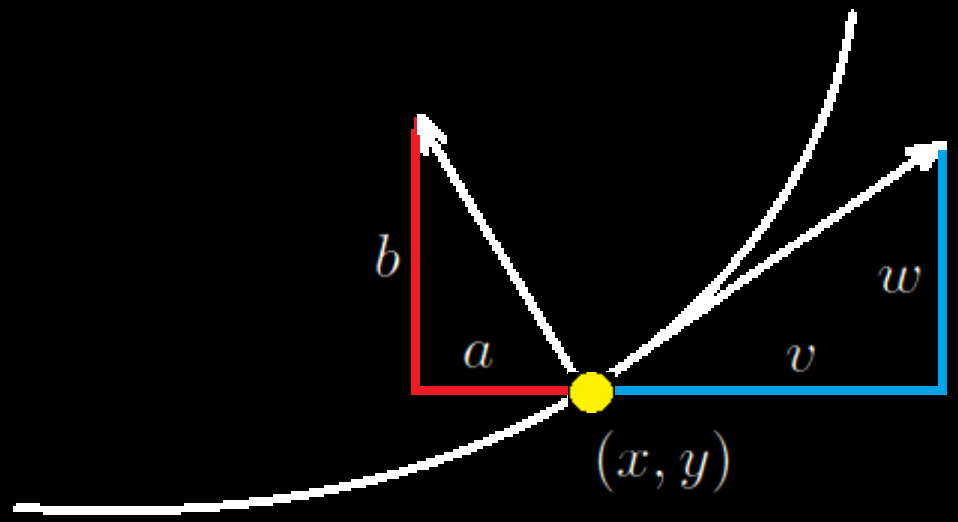
$$\frac{\psi}{m} = \dot{w}$$



Mozgásegyenletek

$$\dot{x}(t) = v(t) \qquad \dot{v}(t) = \frac{\phi(t)}{m}$$

$$\dot{y}(t) = w(t) \qquad \dot{w}(t) = \frac{\psi(t)}{m}$$

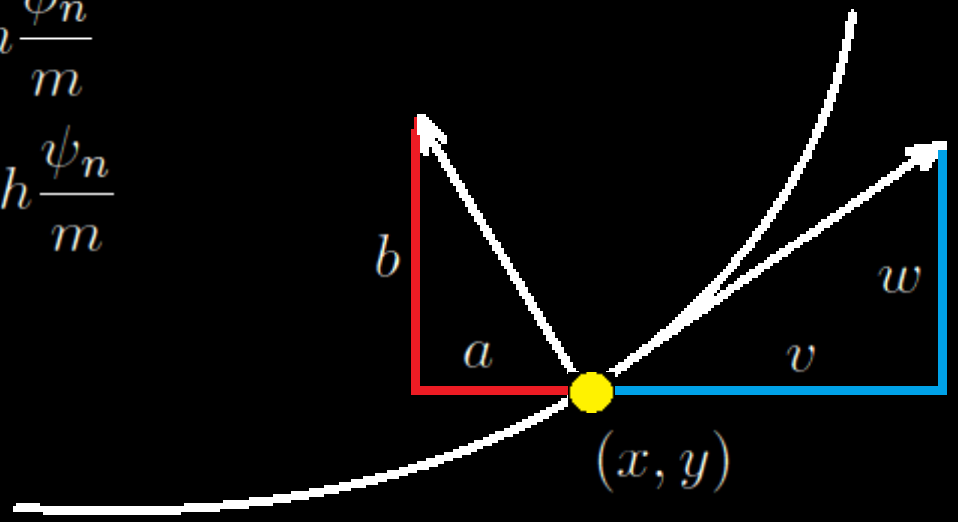


$$\dot{x} = f(t, x)$$

Mozgásegyenletek

$$x_{n+1} = x_n + hv_n \qquad v_{n+1} = v_n + h\frac{\phi_n}{m}$$

$$y_{n+1} = y_n + hw_n \qquad w_{n+1} = w_n + h\frac{\psi_n}{m}$$



$$\dot{x} = f(t,x) \Rightarrow x_{n+1} = x_n + hf(t_n, x_n)$$

Mozgásegyenletek

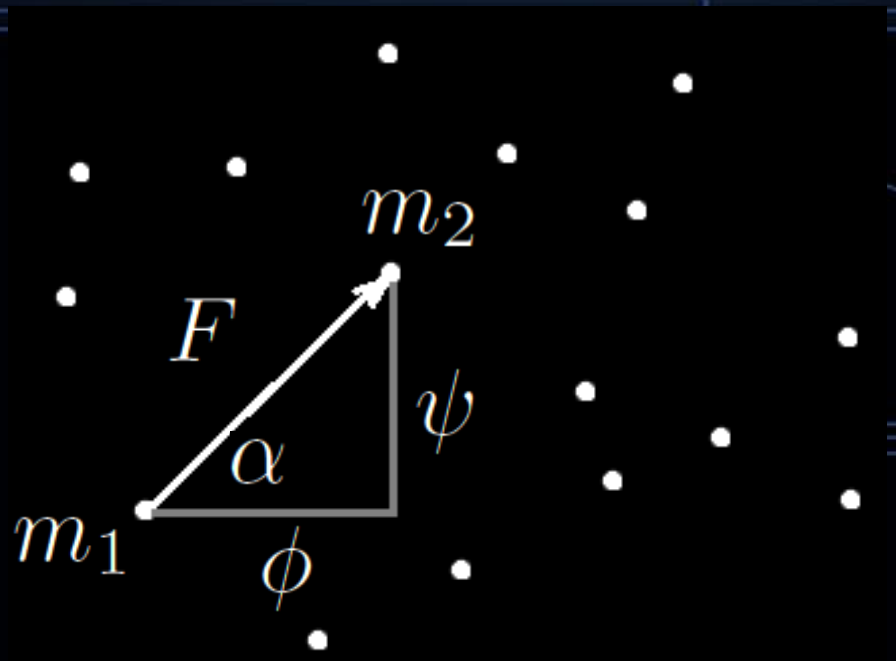
$$x_{n+1} = x_n + h\frac{v_n}{m} \qquad v_{n+1} = v_n + h\frac{\phi_n}{m}$$

$$y_{n+1} = y_n + h\frac{w_n}{m} \qquad w_{n+1} = w_n + h\frac{\psi_n}{m}$$

- Ha tudjuk számolni minden időpillanatban a testre ható erők összegét, ki tudjuk számolni a pályát

Alkalmazás

$$F = G\frac{m_1 m_2}{d_{12}^2}$$



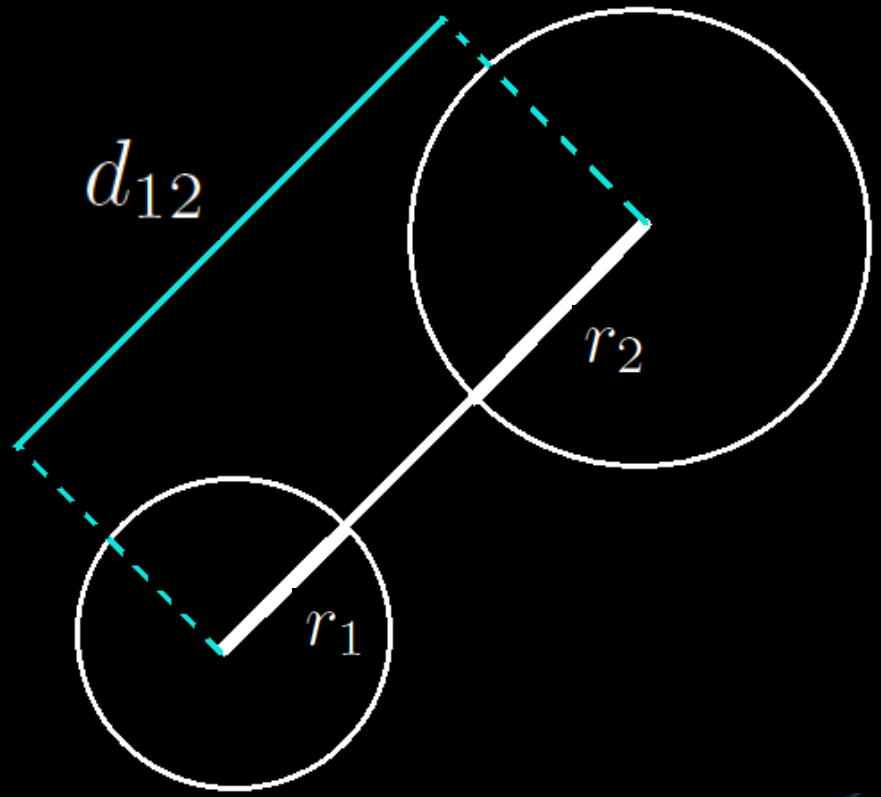
$$\tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

$$d_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Alkalmazás

- Ütközésvizsgálat

$$d_{12} < r_1 + r_2$$



Alkalmazás

```
for l=1:L
    for k=1:drawon
    [x,y,v,w,r,m,X,Y,R]=collision(x,y,v,w,r,m);
    R3=R.^(-3); R3(~isfinite(R3))=0;
    a=(X.*R3)*m; b=(Y.*R3)*m;
    v=v+h*a; w=w+h*b;
    x=x+h*v; y=y+h*w;
    end
    drawstars(x,y,r,m);
end
end
```

```
function [X,Y,R]=distances(x,y)
[xi,xj]=meshgrid(x); X=xi-xj;
[yi,yj]=meshgrid(y); Y=yi-yj;
R=sqrt((abs(X.^2+Y.^2)));
end
```

```
function [x,y,v,w,r,m,X,Y,R]=collision(x,y,v,w,r,m)
[ri,rj]=meshgrid(r); R0=(ri+rj);
[X,Y,R]=distances(x,y);
I=(R<R0).*~eye(size(R));
while sum(sum(I))
    n=length(R); ind=1:n;
    for k=1:n
        if sum(I(:,k))>0
            i=ind(I(:,k)==1); j=ind(I(:,k)==0);
            x(k)=mean(x([k,i])); y(k)=mean(y([k,i]));
            v(k)=mean(v([k,i])); w(k)=mean(w([k,i]));
            m(k)=sum(m([k,i]));
            r(k)=norm(r([k,i]));

            x=x(j); y=y(j);
            v=v(j); w=w(j);
            m=m(j); r=r(j);

            [ri,rj]=meshgrid(r); R0=(ri+rj);
            [X,Y,R]=distances(x,y);
            I=(R<R0).*~eye(size(R));
            break;
        end
    end
end
end
```

Köszönöm a figyelmet

- Videók...