Smooth transition between genus-0 meshes

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Abstract

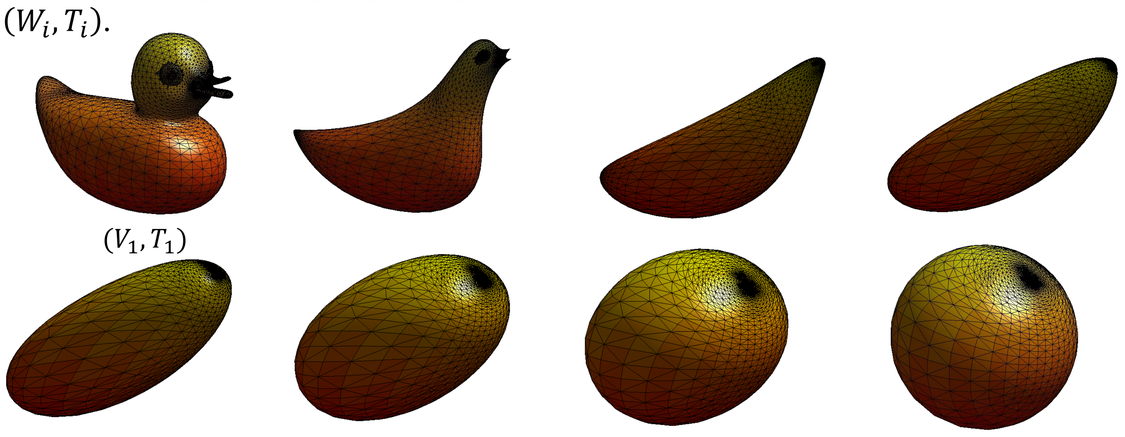
Classification Theorem of Surfaces states, that every orientable closed surface is topologically equivalent to the two-dimensional sphere or the two-dimensional n-tori. Topological equivalence of two surfaces X and Y means that there exists a homeomorphism between X and Y, i.e. there is an $H: X \to Y$ function, that is bijection, continuous and inverse continuous. The concept of homeomorphism can be thought as a **continuous deformation**, roughly speaking we can say, surfaces are topologically equivalent, if they can be deformed into each other without cutting and gluing. In our research we deal with **triangular meshes**, that are the most common representation of surfaces in computer graphics. By **Euler's Theorem** it can be easily verified that a triangular mesh is topologically equivalent to the sphere, but defining a homeomorphism between them is a difficult task. Fortunately, there can be found several algorithms in literature, that are able to solve this problem. As the composition of homeomorphisms is homeomorphism, if the surfaces X and Y both are topologically equivalent to the sphere, we know that there exist a homeomorphism that sends the points of X to Y. In this research we are interested in finding a homeomorphism between two meshes, that are equivalent to the sphere. We are using simple, known algorithms as building blocks to develop an easy-to-implement easily understandable solution of the problem.

INTRODUCTION

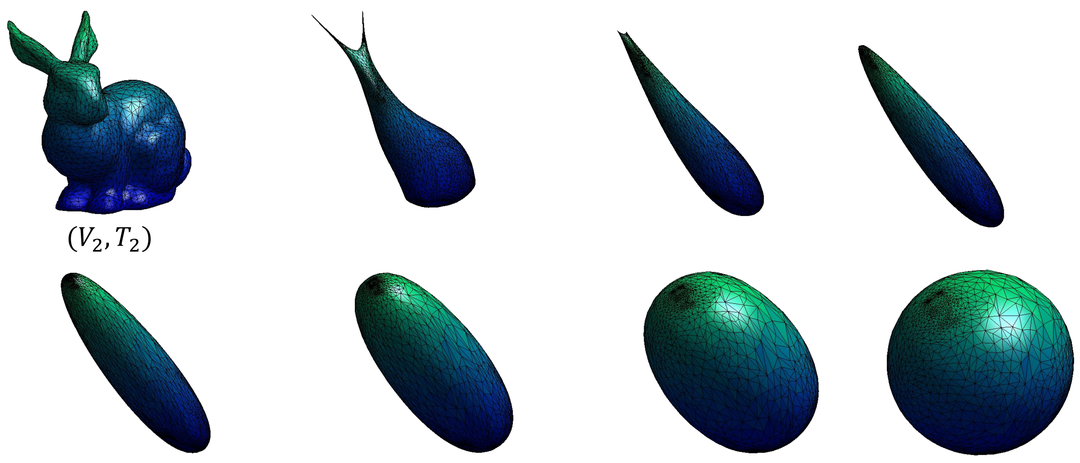
Let us consider two triangular meshes defined by (V_1, T_1) and (V_2, T_2), where V_i $(i \in \{1,2\})$ contain vertex positions, T_i contain index triplets defining surface elements spanned by the corresponding vertex triplets. The topological equivalence can be checked by counting the number of vertices(m), edges (l) and triangles (n). Euler's theorem implies, that a (well-defined) mesh is topologically equivalent to the sphere, if $m - l + n = 2$. The surfaces having this property are called genus-0 surfaces. Generally the number of vertices and triangles are different in two topologically equivalent meshes, our strategy is to create a „supermesh", that contains both (V_1, T_1) and (V_2, T_2), therefore a smooth transition between them can be obtained by moving the vertices of the supermesh from one position to another.

RESULTS

First we use a so-called mean curvature flow to send the vertices in V_i to the unit sphere without causing overlapping triangles in T_i. Let us denote the transformed meshes by (W_i, T_i).

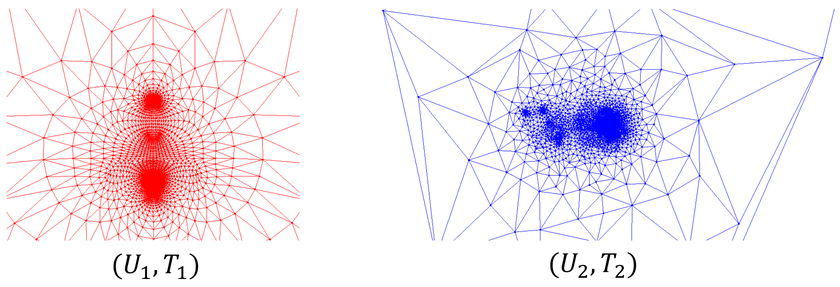


(V_1, T_1) (W_1, T_1)



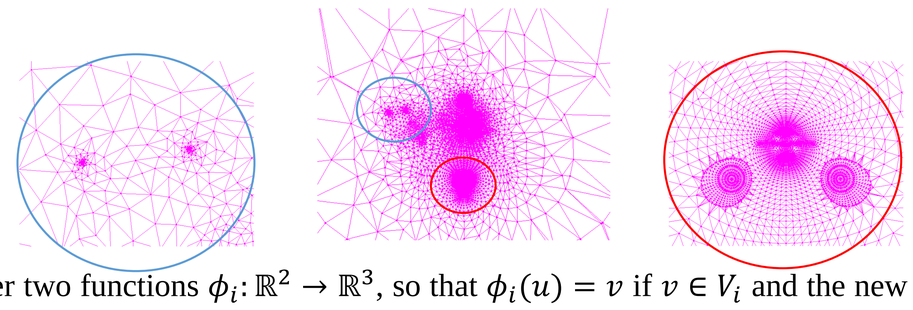
(V_2, T_2) (W_2, T_2)

After that, we calculate the stereographic projection of vertices of W_i to obtain the two-dimensional vertex sets U_i.

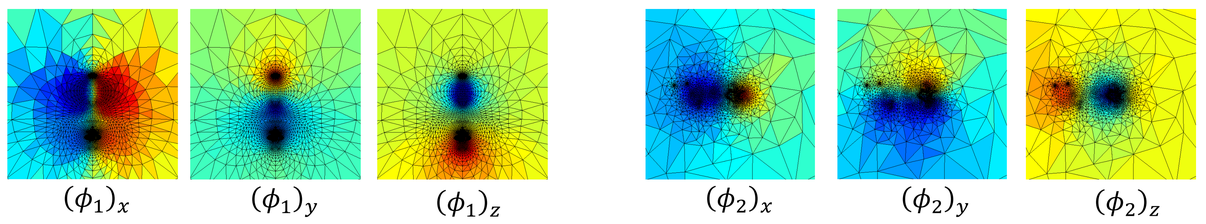


(U_1, T_1) (U_2, T_2)

Now, the union of vertex sets $U = U_1 \cup U_2$ contains all the points, that are sufficient to reconstruct both of the meshes.

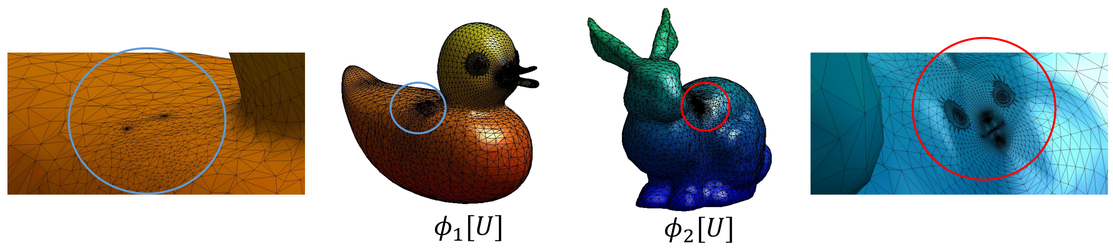


Consider two functions $\phi_i: \mathbb{R}^2 \to \mathbb{R}^3$, so that $\phi_i(u) = v$ if $v \in V_i$ and the new position of u after the mean curvature flow and the stereographic projection is u. This case ϕ_i functions are the three-dimensional interpolants of scattered data points in U_i. For simplicity, consider the coordinate functions of ϕ_i.



$(\phi_1)_x$ $(\phi_1)_y$ $(\phi_1)_z$ $(\phi_2)_x$ $(\phi_2)_y$ $(\phi_2)_z$

They are simple scalar valued functions, therefore $\phi_i(x)$ can be defined for all $x \in \mathbb{R}^2$ using common 2D interpolation and extrapolation techniques. After that, values of ϕ_i are calculated in all points of U, and the T triangulation of the point set is determined. Then the original (V_i, T_i) meshes can we get back from $(\phi_i[U], T)$.



$\phi_1[U]$ $\phi_2[U]$

The significant difference is that the index sets of the two meshes are the same, consequently triangles can be pairwise assigned to each other. Finally we use linear interpolation between the vertex positions $(\phi_1[U], T)$ and $(\phi_2[U], T)$ to obtain a smooth transition between the meshes.

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