How to interpret Meshes as Images

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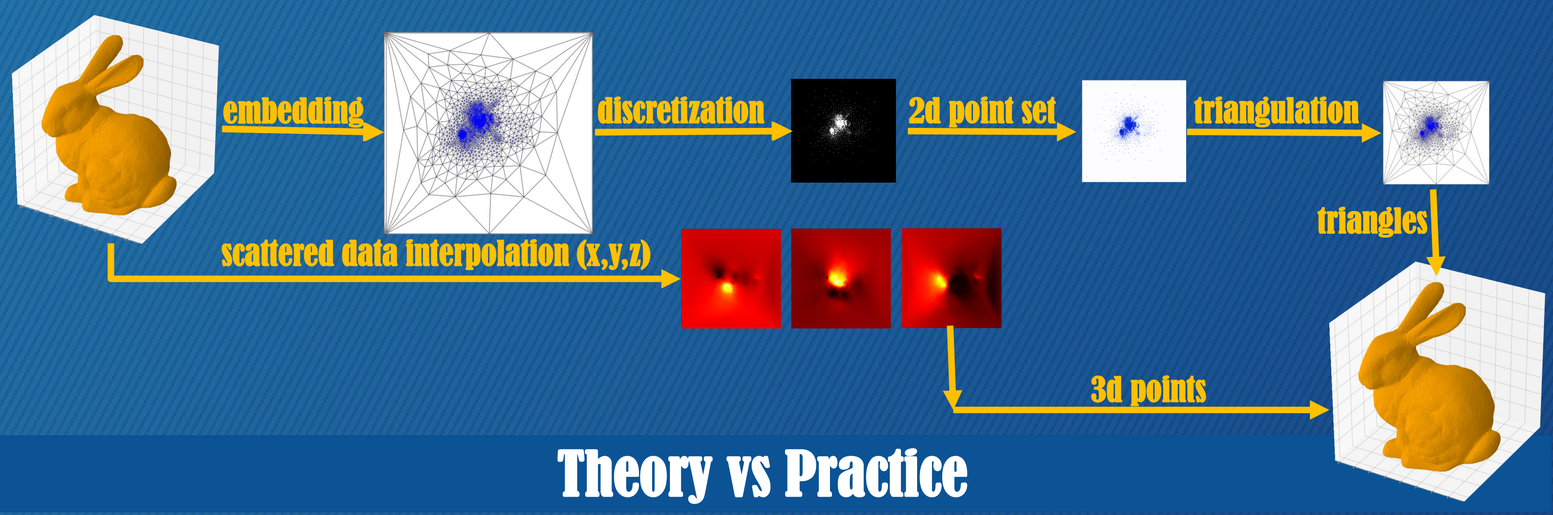
Abstract

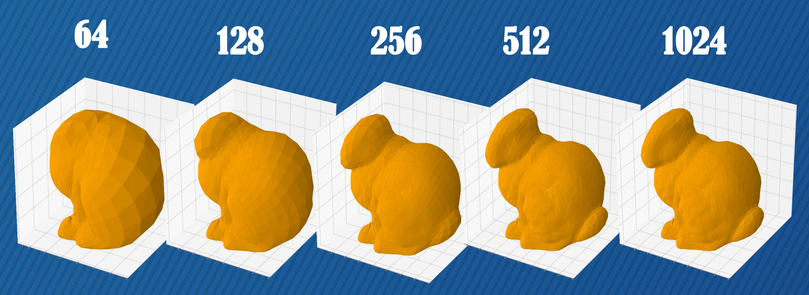
In computer graphics the most common representation of a surface is the so-called triangle mesh. The triangle mesh can be imagined as a 3-connected graph embedded into the 3-dimensional space. In contrast, in geometry the surfaces are 3-dimensional point sets, that can be mapped onto a 2-dimensional interval by a bijection. These types of surfaces are called parametric surfaces, the bijection is called a parametrization of the surface [1]. Mesh parametrization is the problem of finding such a parametrization of a triangle mesh. Knowing the parametrization of a mesh can be useful in many ways, e.g. by detail mapping, mesh editing, remeshing, mesh compression, mesh morphing, etc. [4] On the other hand it is easy to see, that the coordinate functions of the parametrization can be considered to be (grayscale) images. This way we can think of surfaces as images, we can store, compress and process them as images. Commonly finding a parametrization of a mesh is not an easy task, but if the mesh is a triangulation of a surface which is topologically equivalent to the sphere, it is getting easier. Tutte's embedding theorem states, these types of graphs (as well as meshes) can be drawn on the plane such a way, that its edges are straight line segments that do not cross each other. [2] [6] This mapping is often called straight line embedding or Tutte-embedding. Using the Tutte-embedding we can easily construct a parametrization of a mesh, for instance by determining the proper linear interpolant of the inverse of the embedding [5]. Furthermore, if we have a parametrization, we can define a new point set on the plane, which can be easily triangulated [3], after that applying the inverse of the embedding to this triangulation, an approximation of the original surface can be obtained, which is the base idea behind the remeshing and (lossy) mesh compression. Although Tutte's embedding theorem gives a surprisingly easy method to find an embedding of a graph, it can be criticized due to the fact, that the resulting graph is unbalanced, i.e. in the „inner regions" the vertices are located much more densely than „close to the periphery". On the other hand this type of embedding is useful, since it can be determined by solving only a sparse linear equation system, maybe this is the easiest way to find a parametrization of a mesh.
In our research we show, how to use Tutte's embedding for finding a parametrization of a mesh, how to refine the result to obtain a graph, in which the geometric information is homogeneously distributed. In our work the following technique was used. Firstly a bijective mapping of the two-dimensional space onto itself was iteratively used to increase the distance between the points of the parametrization. After that a non-uniform grid was used to discretize the continuous domain. As we show, using these intermediate processing steps can increase the accuracy of the reconstruction.

Theoretical approach

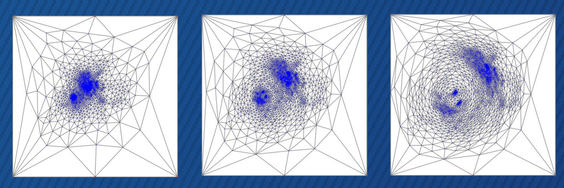


Theory vs Practice

What can we do, when refinement of the discretization is not enough?



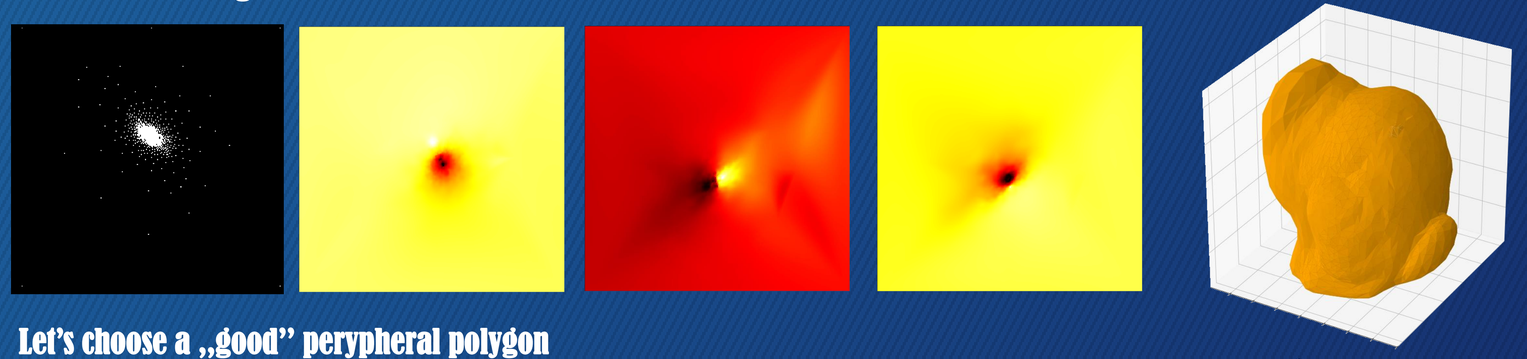
Space warps



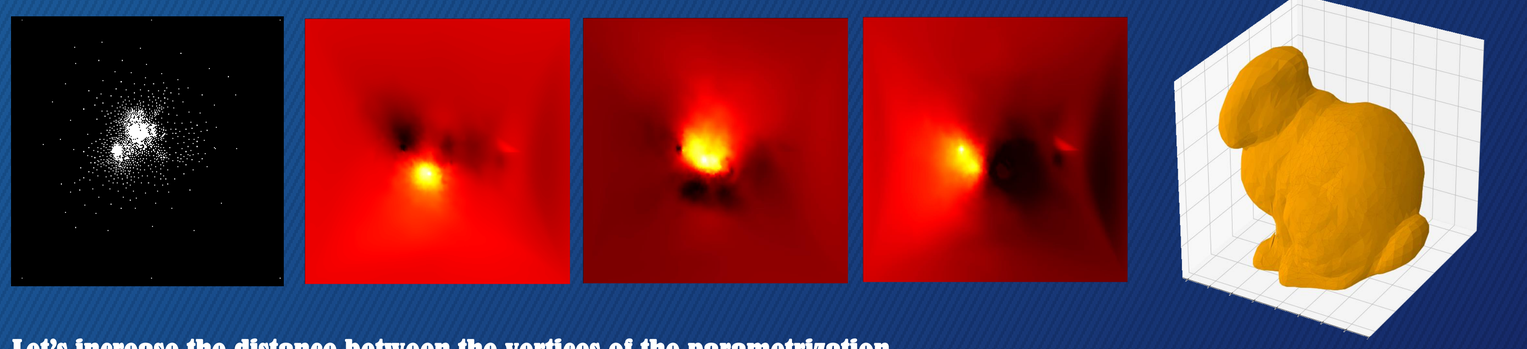
Non-uniform discretization

Distribute the geometric information

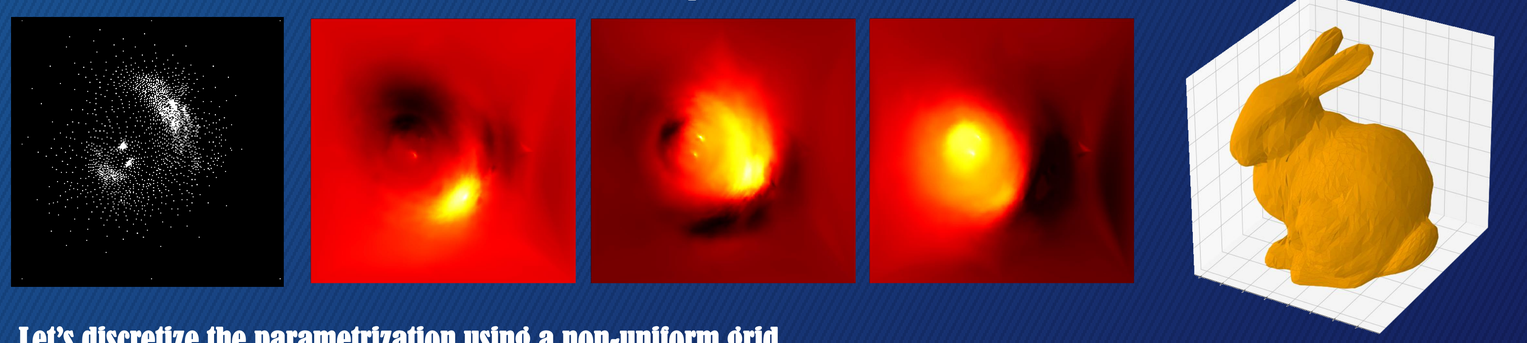
Let's do something



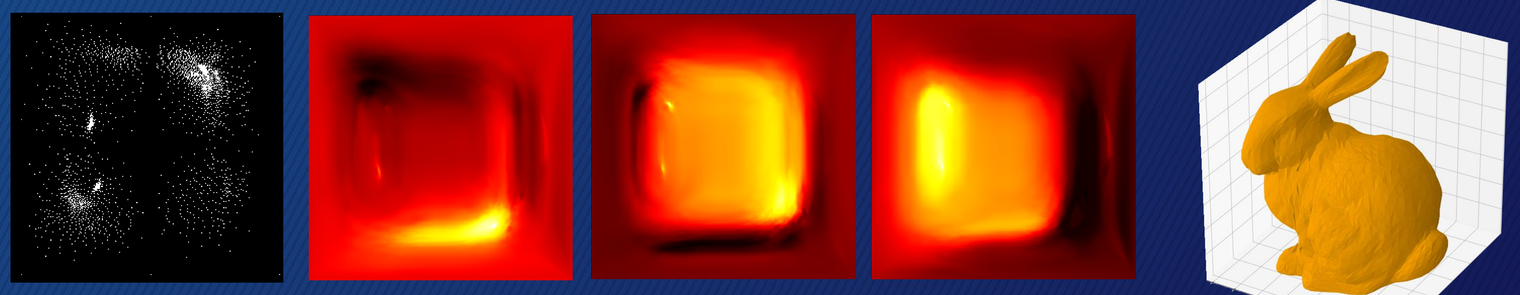
Let's choose a „good" perypheral polygon



Let's increase the distance between the vertices of the parametrization



Let's discretize the parametrization using a non-uniform grid



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