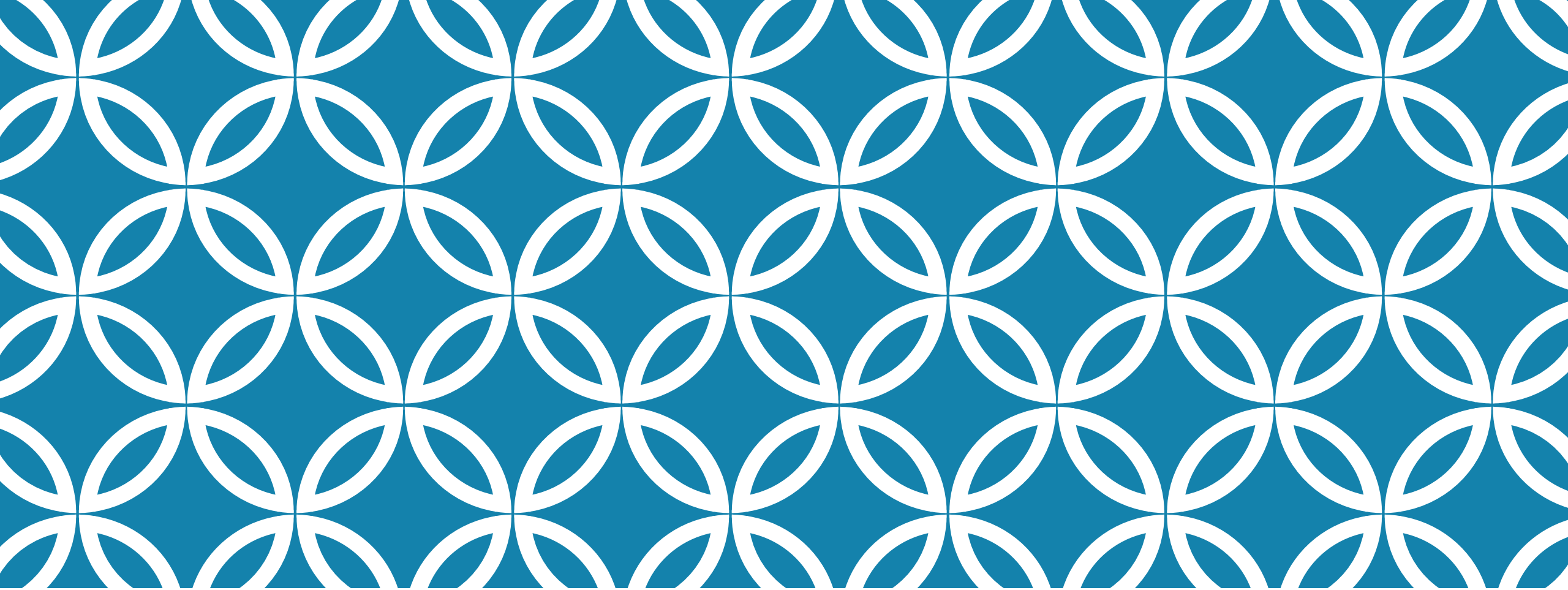


FORGATÁSOK - AZ EUKLIDESZI GEOMETRIÁTÓL A ZENEI UJJLENYOMATIG

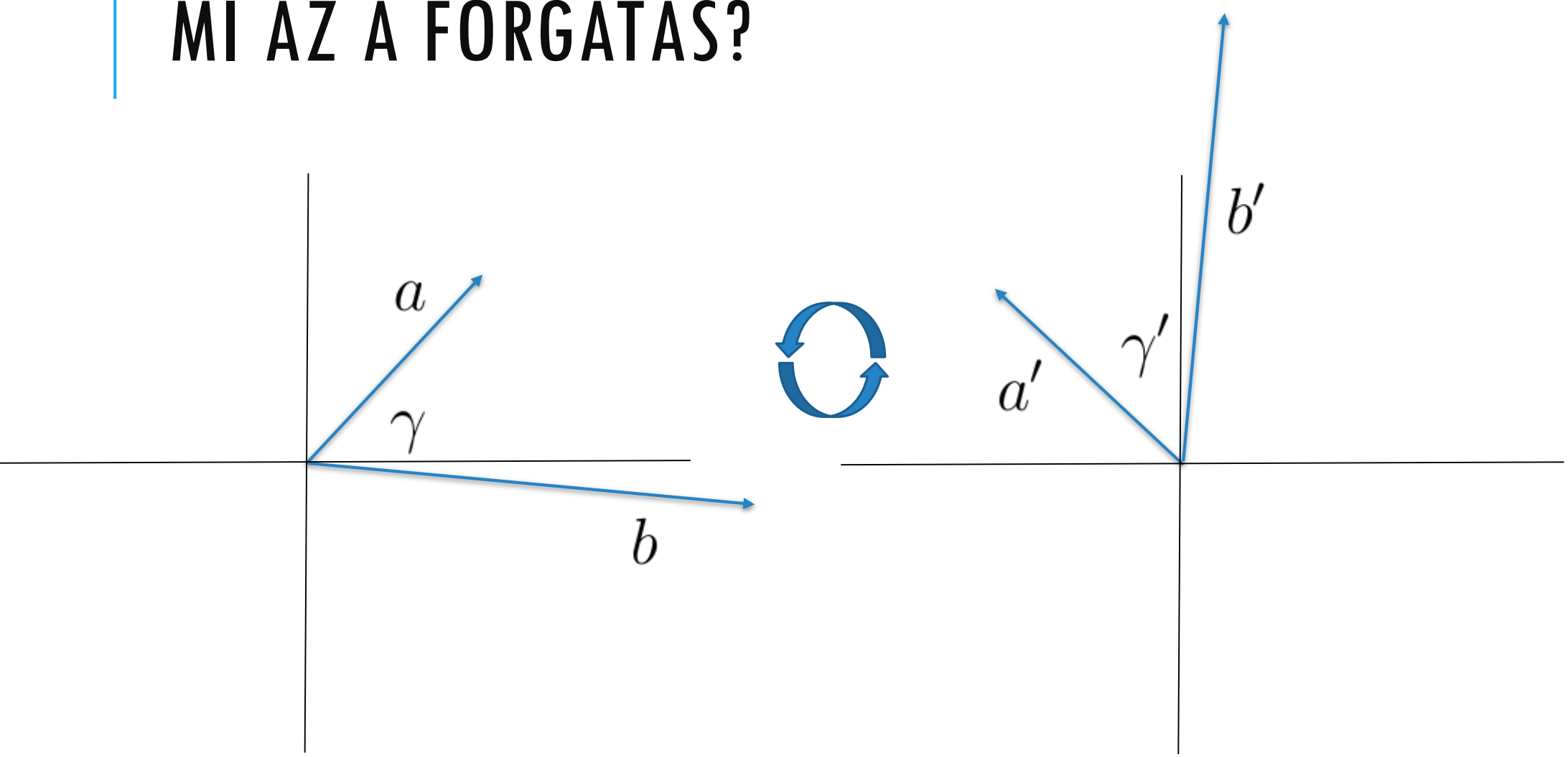
Fábián Gábor
ELTE-IK
Numerikus Analízis Tanszék
2015. 05. 02.
XX. Bolyai konferencia, Budapest

GEOMETRIA

Alexandriai Euklidész (Kr.e. 300 körül)
Στοιχεῖα
http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf

David Hilbert (1862-1943)
Grundlagen der Geometrie (1899)
https://archive.org/details/grunddergeovon00hilbrich

MI AZ A FORGATÁS?



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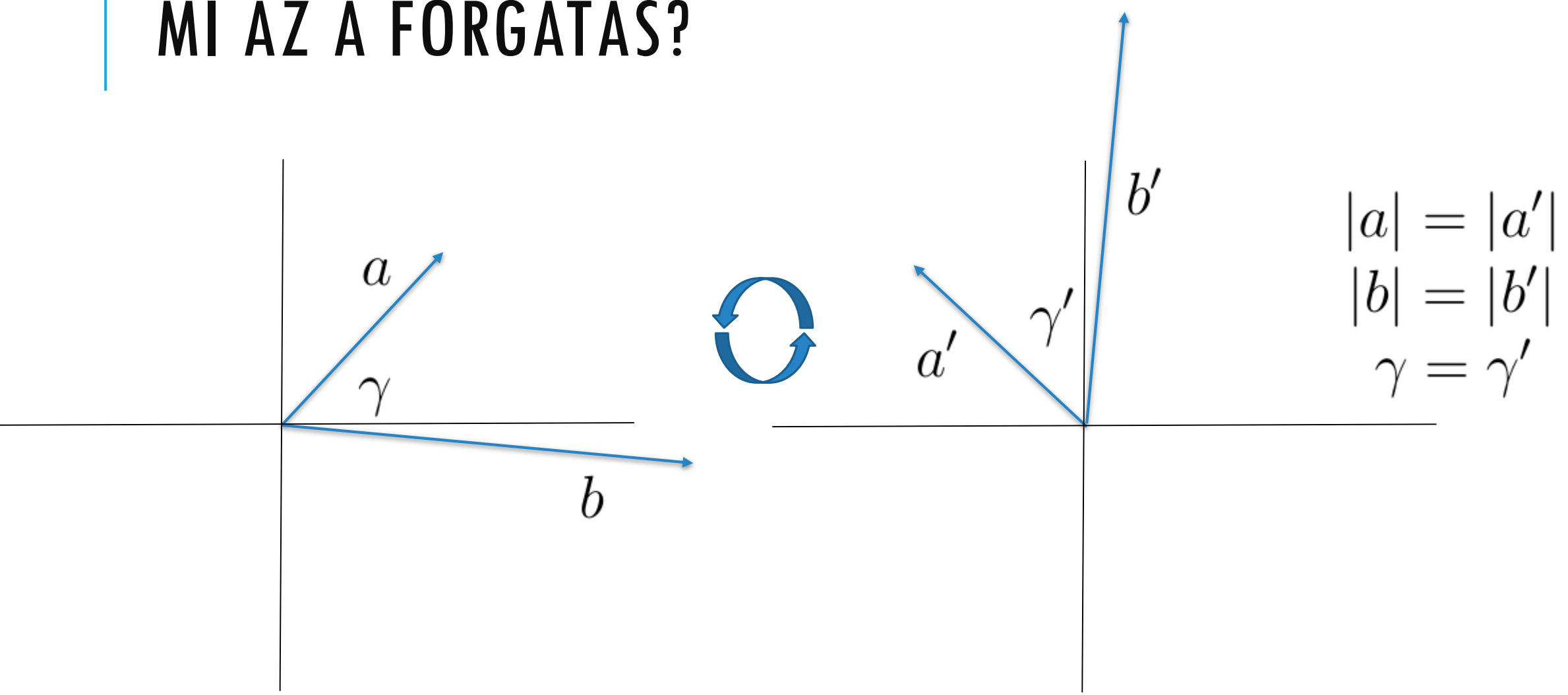

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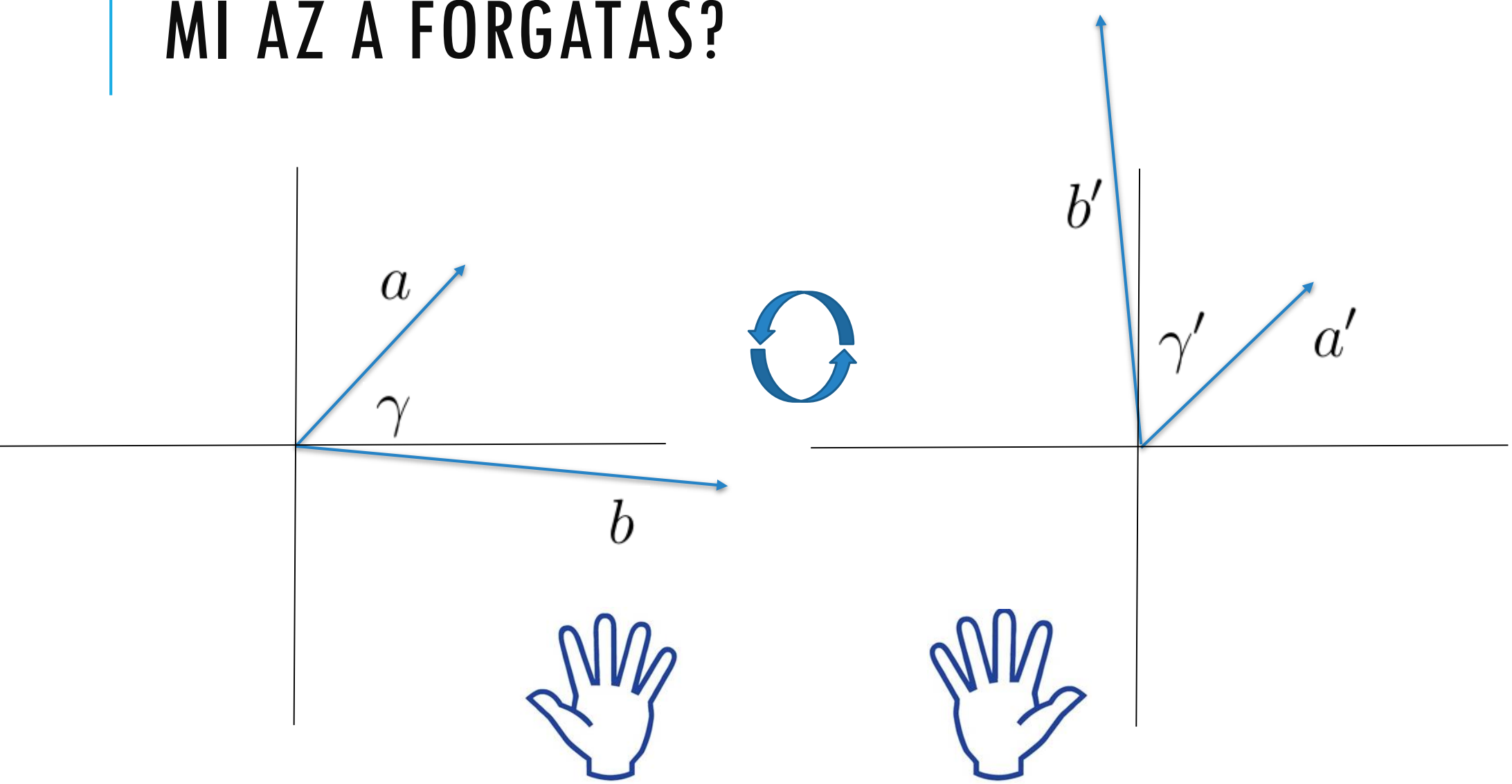

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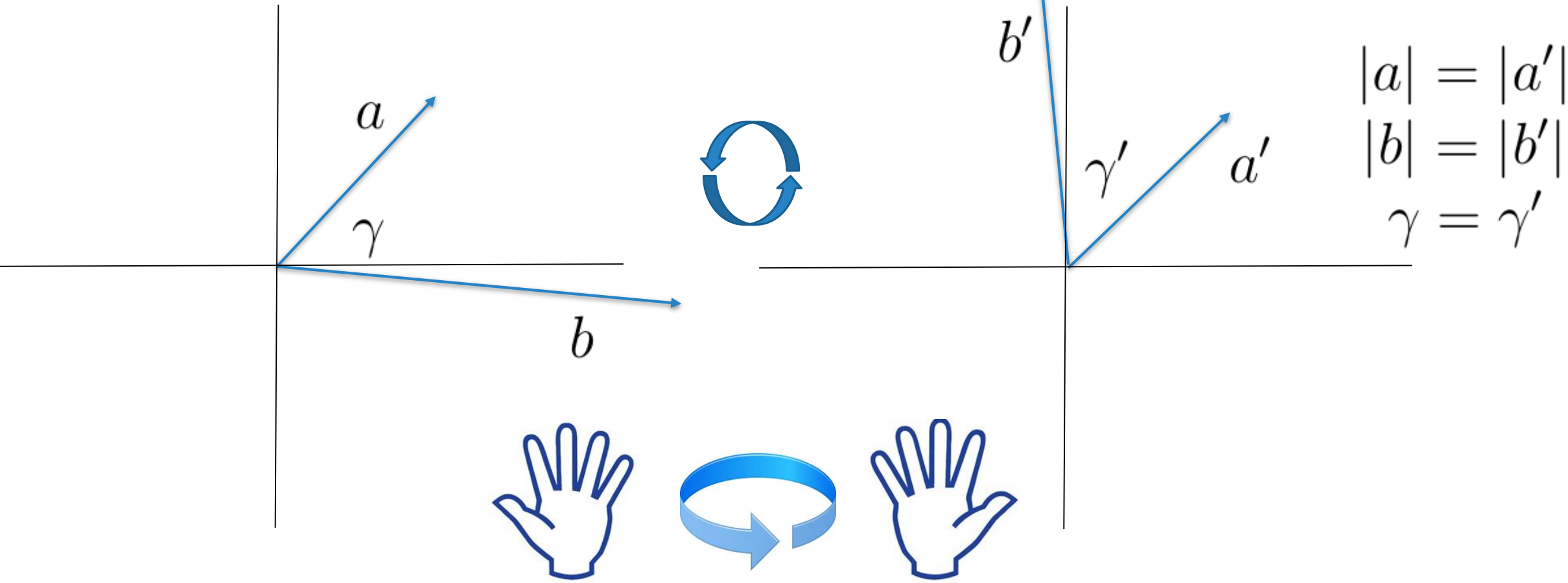
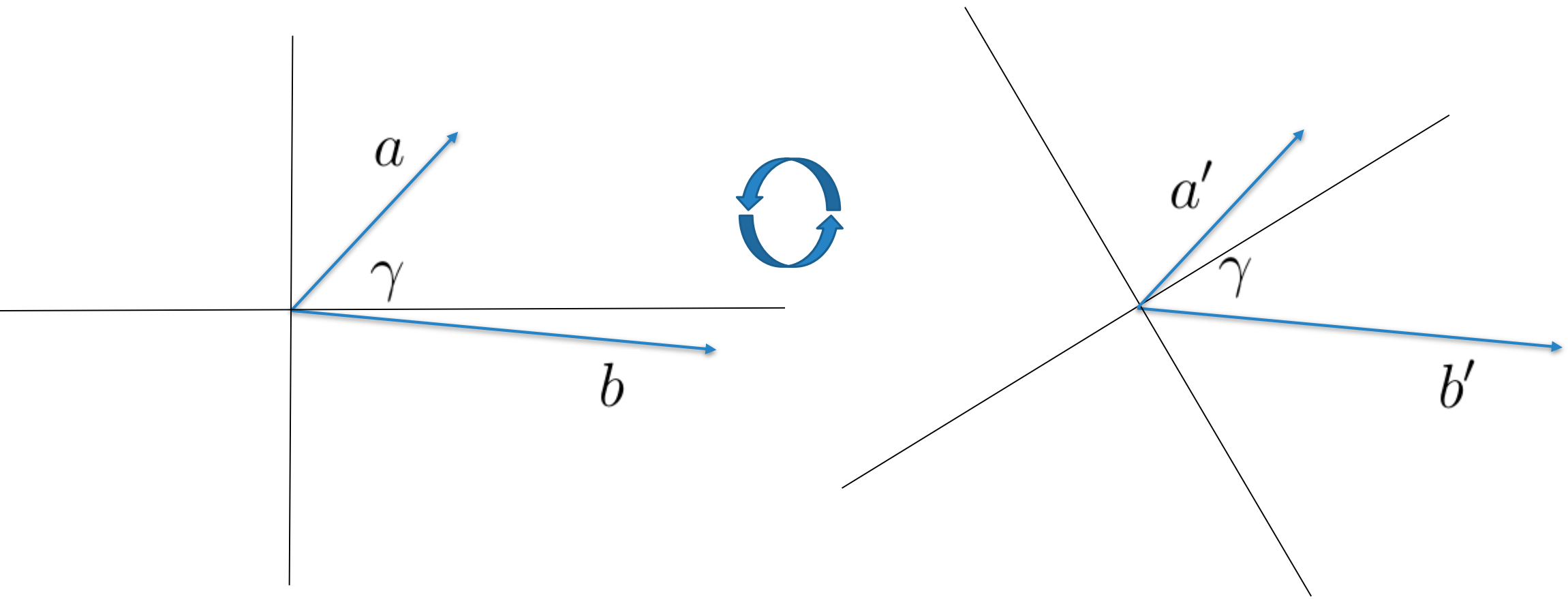
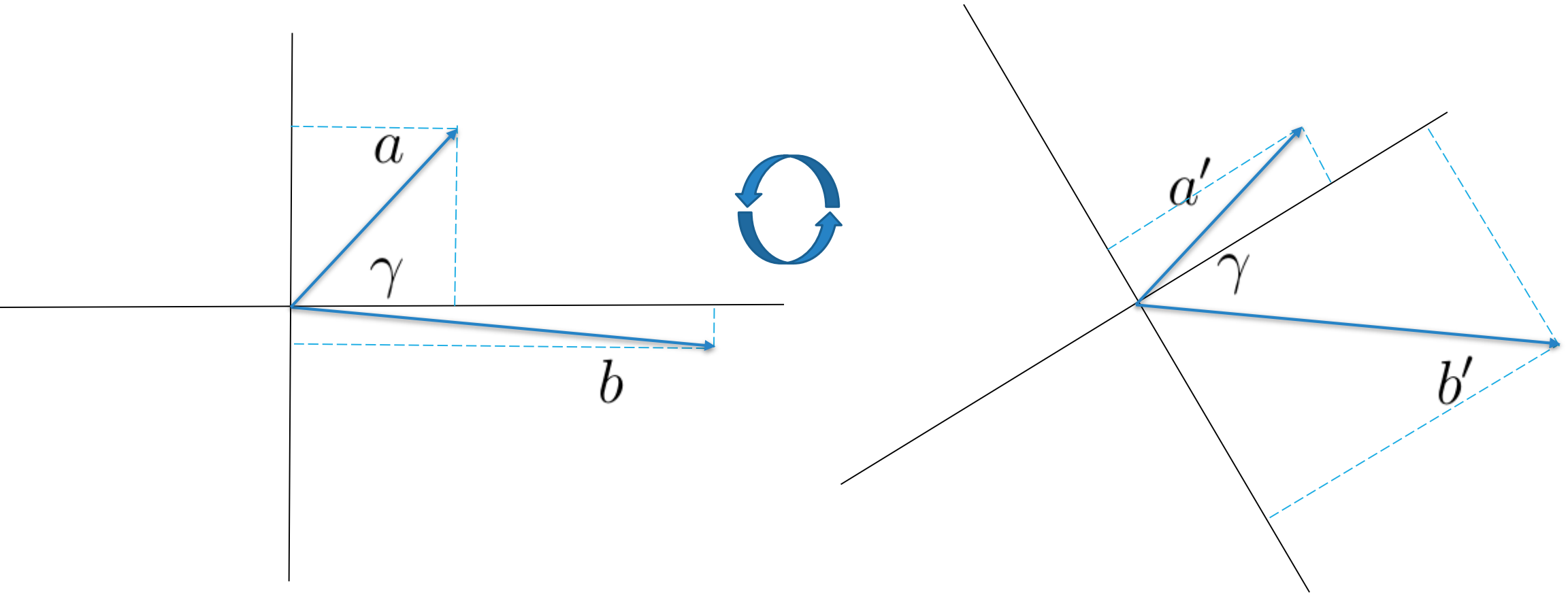


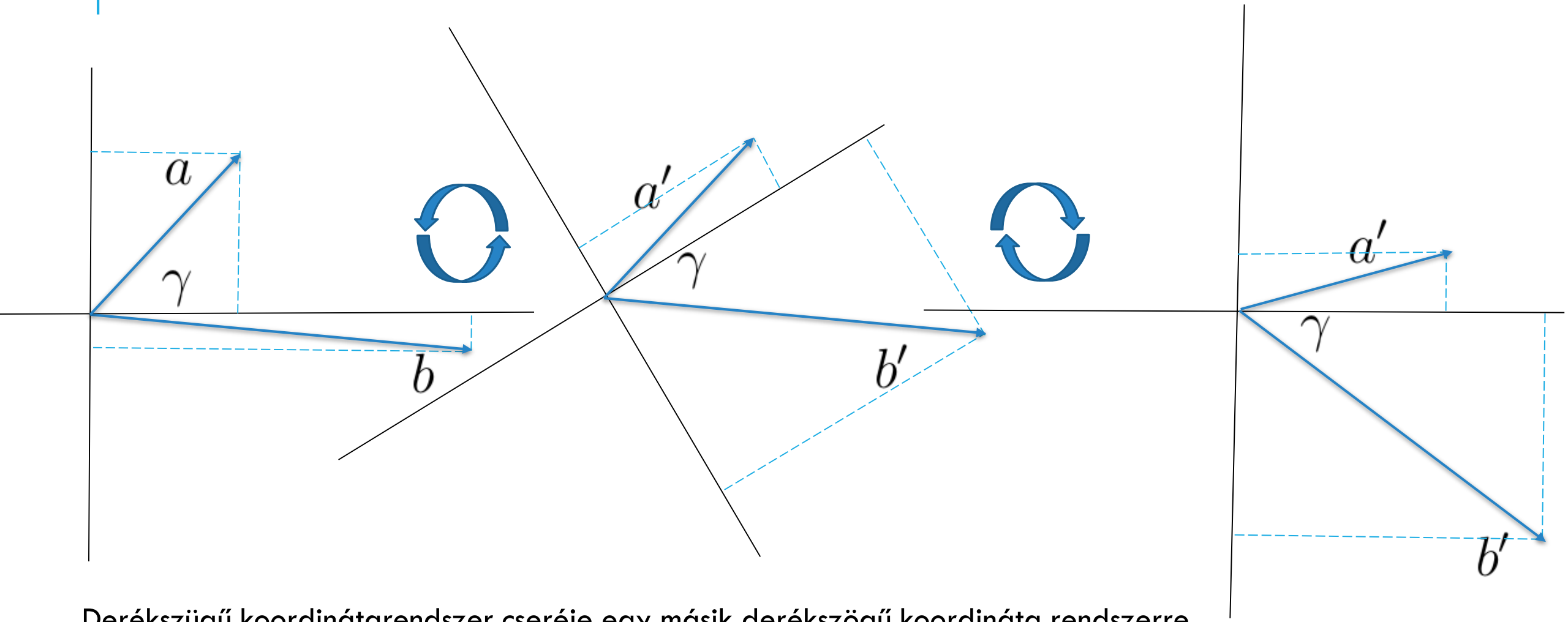
HOGYAN FORGATUNK?



MERŐLEGES VETÍTÉS

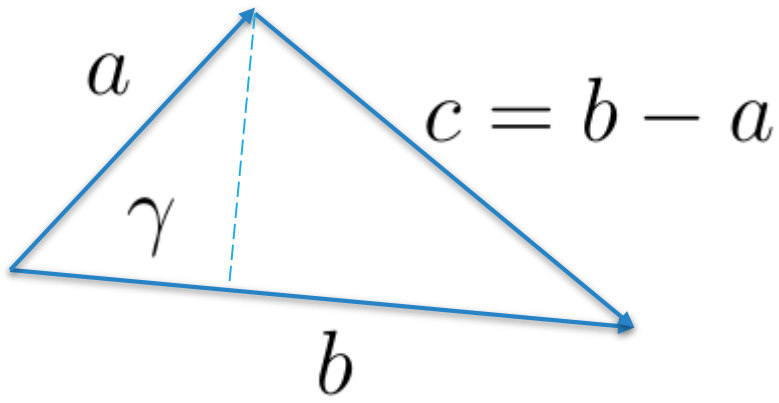


TEHÁT A FORGATÁS



Derékszügű koordinátarendszer cseréje egy másik derékszögű koordináta rendszerre

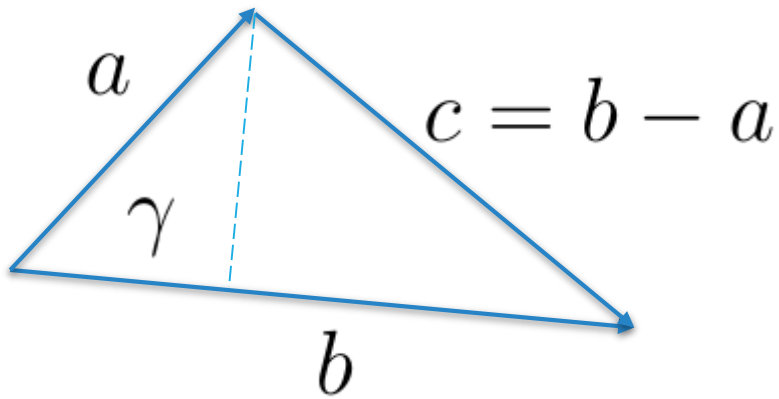
MERŐLEGES VETÍTÉS

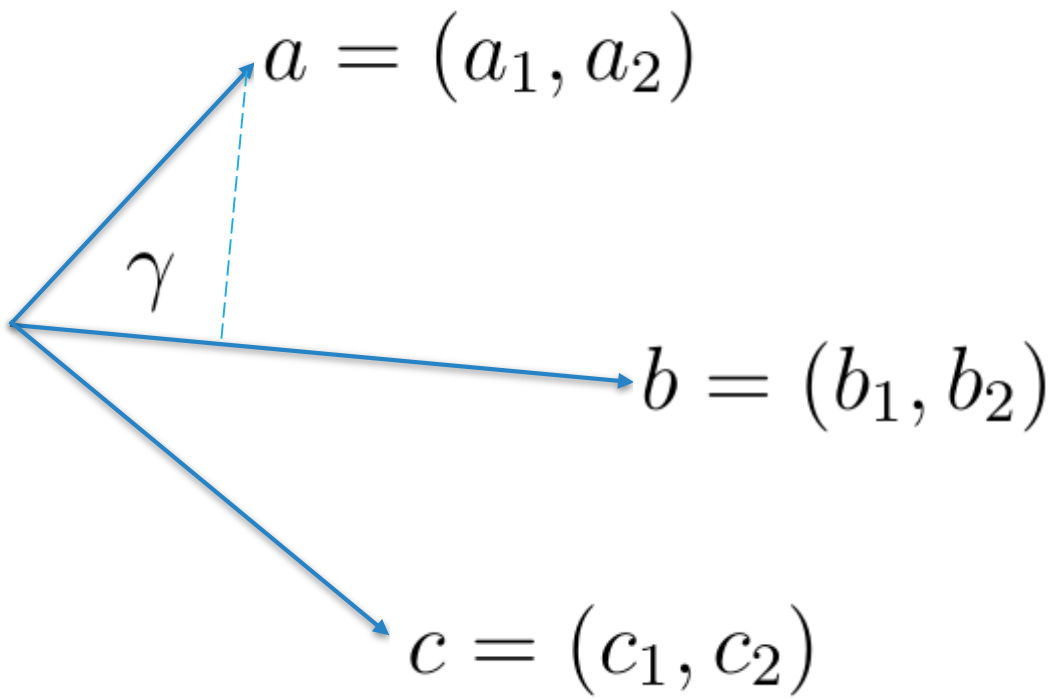


MERŐLEGES VETÍTÉS

$$\langle a, b \rangle = a_1 b_1 + a_2 b_2 = \sum_{i=1}^{2} a_i b_i$$

$$|c|^2 = c_1^2 + c_2^2 = \sum_{i=1}^{2} c_i^2$$





SKALÁRSZORZAT

$$a = (a_1, a_2) \qquad b = (b_1, b_2) \qquad c = (c_1, c_2)$$

$$|c|^2 = |a|^2 + |b|^2 - 2|a||b|\cos\gamma$$

$$|c|^2 = c_1^2 + c_2^2 = \sum_{i=1}^{2} c_i^2 = \sum_{i=1}^{2} (b_i - a_i)^2 = \sum_{i=1}^{2} b_i^2 - 2b_i a_i + a_i^2 = |a|^2 + |b|^2 - 2\langle a, b\rangle$$

$$-2|a||b|\cos\gamma = -2\langle a, b\rangle$$

$$|a||b|\cos\gamma = \langle a, b\rangle$$

SKALÁRSZORZAT

$$a = (a_1, \ldots, a_n) \qquad b = (b_1, \ldots, b_n) \qquad c = (c_1, \ldots, c_n)$$

$$|c|^2 = |a|^2 + |b|^2 - 2|a||b| \cos\gamma$$

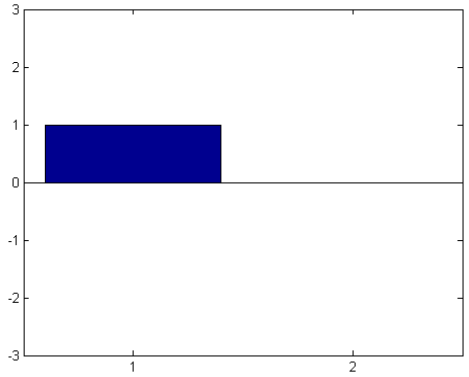
$$|c|^2 = c_1^2 + \cdots + c_n^2 = \sum_{i=1}^{n} c_i^2 = \sum_{i=1}^{n} (b_i - a_i)^2 = \sum_{i=1}^{n} b_i^2 - 2b_i a_i + a_i^2 = |a|^2 + |b|^2 - 2\langle a, b\rangle$$

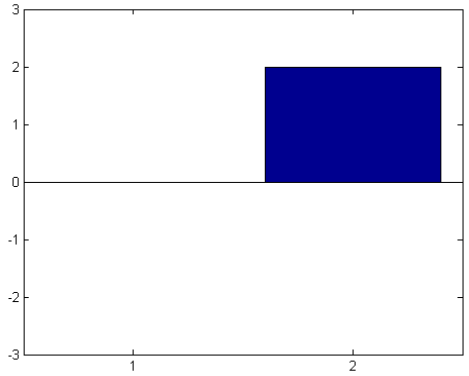
$$-2|a||b| \cos\gamma = -2\langle a, b\rangle$$

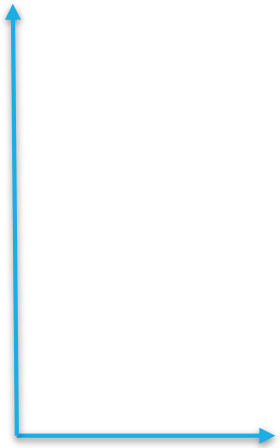
$$|a||b| \cos\gamma = \langle a, b\rangle$$

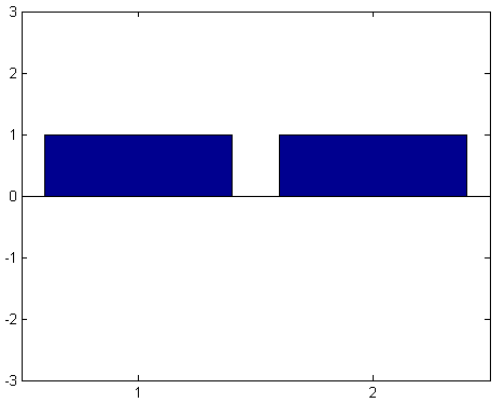
y

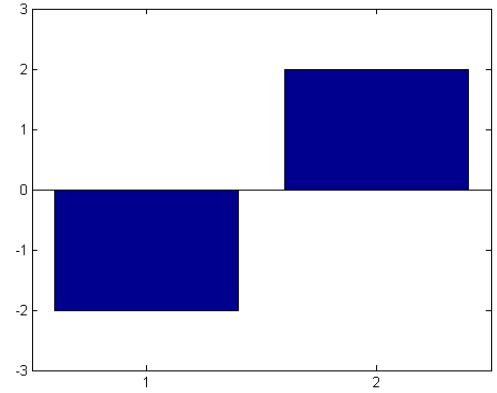
AZ N DIMENZIÓS TÉR

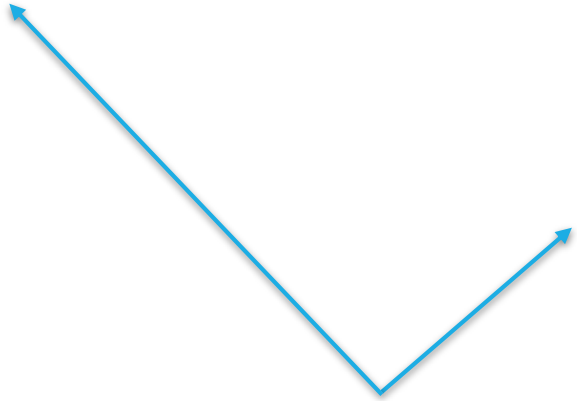






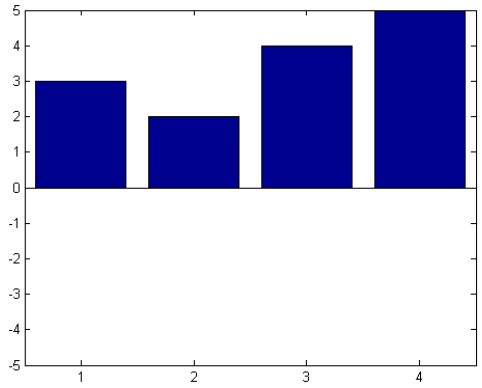


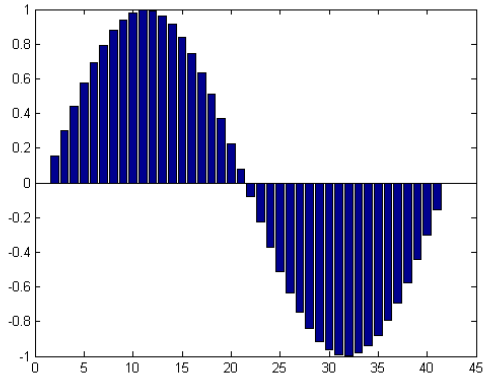


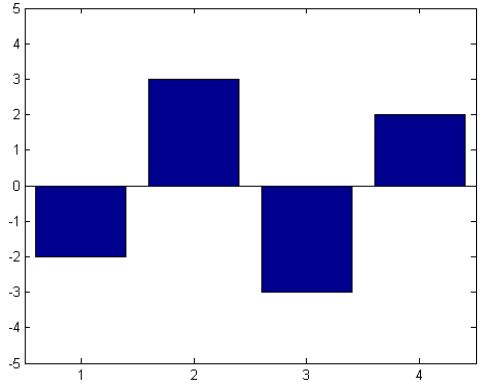


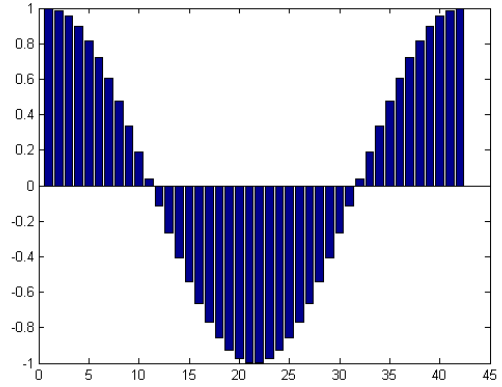
x

AZ N DIMENZIÓS TÉR









FORGATÁS

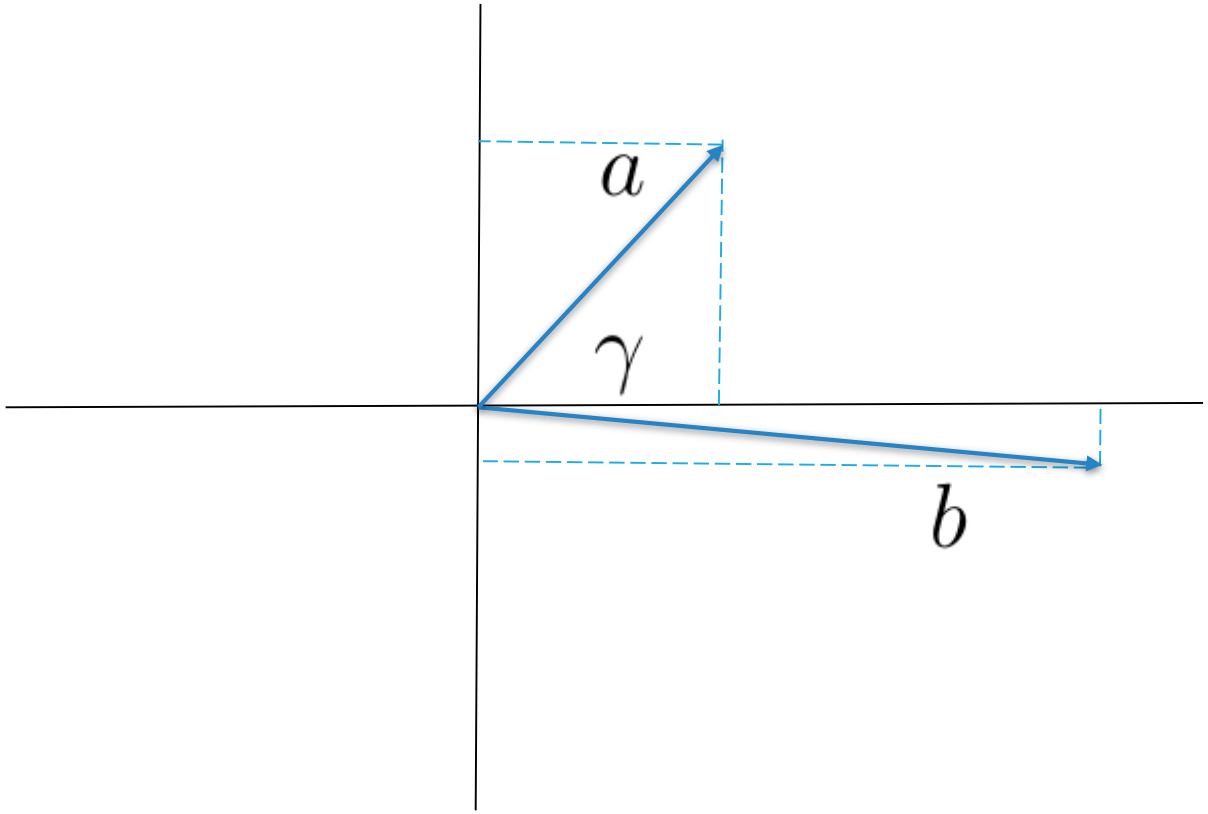
$$a = (a_1, \ldots, a_n)$$

$$e^{(i)} = (0, \ldots, 0, 1, 0, \ldots, 0)$$

(1 at position i)

$$a_i = \langle a, e^{(i)} \rangle$$

$$a = \sum_{i=1}^{n} \langle a, e^{(i)} \rangle e^{(i)}$$



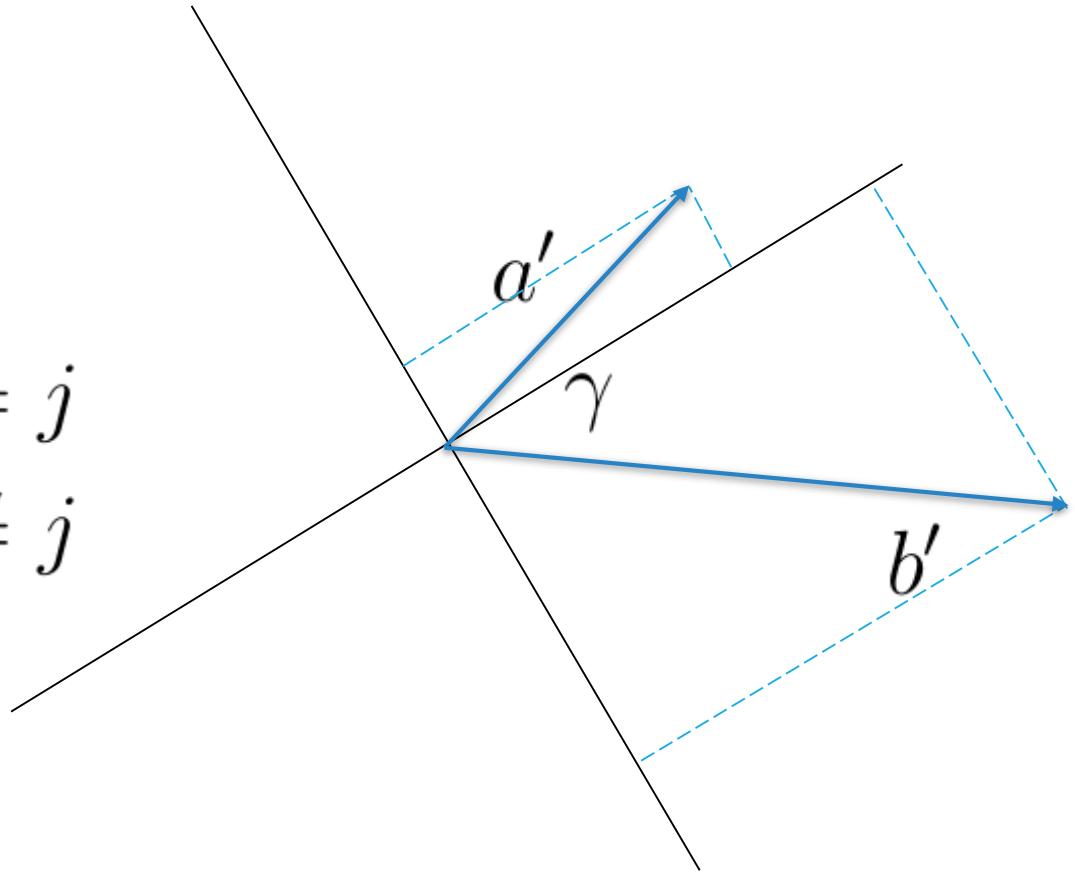
FORGATÁS

$$a = (a_1, \ldots, a_n)$$

$$b^{(i)} = (b_1^{(i)}, \ldots, b_n^{(i)})$$

$$\langle b^{(i)} b^{(j)} \rangle = \sum_{k=1}^{n} b_k^{(i)} b_k^{(j)} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$$a = \sum_{i=1}^{n} \langle a, b^{(i)} \rangle b^{(i)}$$



A FUNKCIONÁLANALÍZIS SZÜLETÉSE

$$\sum_{i=1}^{n} a_i b_i \qquad \sum_{i=1}^{\infty} a_i b_i \qquad \sum_{i=1}^{\infty} f(x_i) g(x_i) \Delta x_i \qquad \int f(x) g(x) \mathrm{d}x$$

Riesz Frigyes
(1880-1956)

FOURIER-TRANSZFORMÁCIÓ

Jean Baptiste Joseph Fourier (1768-1830)

$$\frac{\partial u}{\partial t} = \alpha \Delta u$$

FOURIER-TRANSZFORMÁCIÓ

Jean Baptiste Joseph Fourier (1768-1830)

$$\frac{\partial u}{\partial t} = \alpha \Delta u$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

FOURIER-TRANSZFORMÁCIÓ

Jean Baptiste Joseph Fourier (1768-1830)

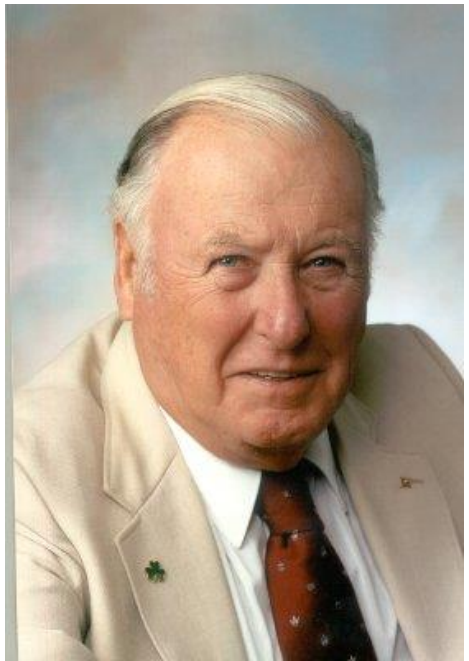
$$\frac{\partial u}{\partial t} = \alpha \Delta u$$

FAST FOURIER TRANSFORM

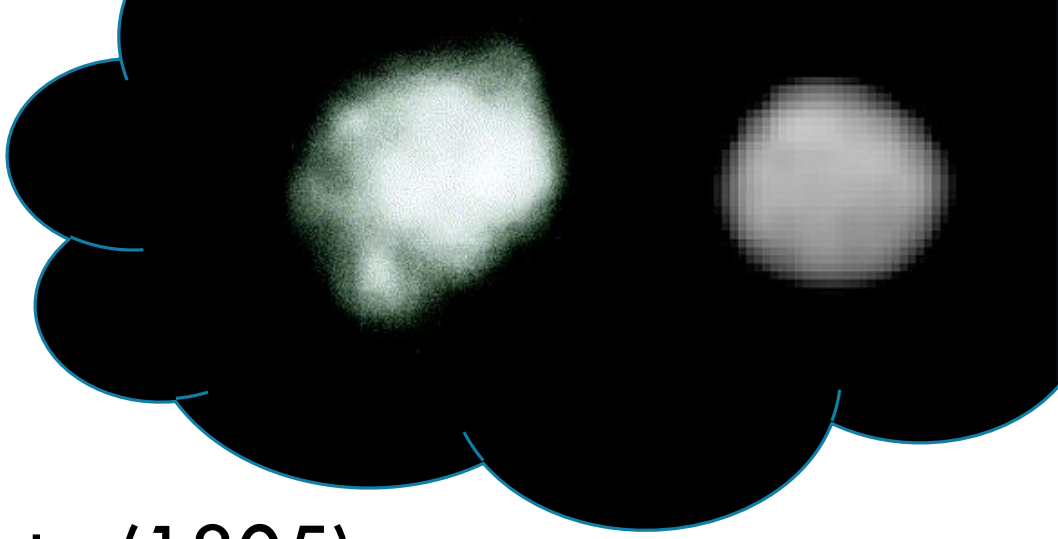
An algorithm for the machine calculation of complex Fourier series (1965)
https://web.stanford.edu/class/cme324/classics/cooley-tukey.pdf

John Wilder Tukey
(1915-2000)

James William Cooley
(1926-)



FAST FOURIER TRANSFORM

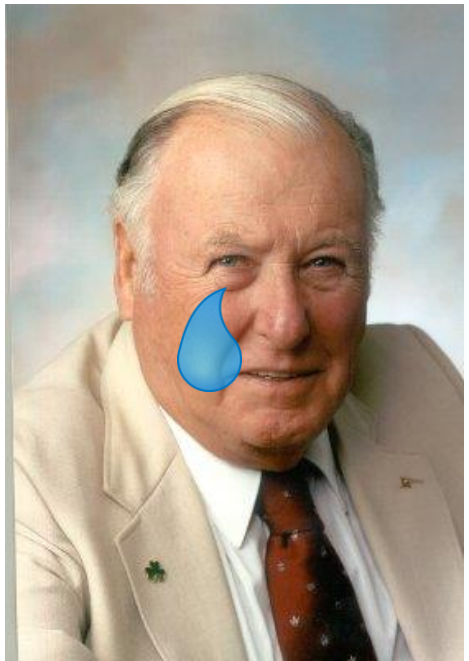


Theoria interpolationis methodo nova tractata (1805)

John Wilder Tukey
(1915-2000)

Carl Friedrich Gauss
(1777-1855)

James William Cooley
(1926-)



FAST FOURIER TRANSFORM

FFT $n \log n$

DFT n^2

FAST FOURIER TRANSFORM

	n	10^9
FFT	$n \log n$	$2.7 \cdot 10^{10}$
DFT	n^2	10^{18}

FAST FOURIER TRANSFORM

	n	1000000000
FFT	$n \log n$	27000000000
DFT	n^2	1000000000000000000

MENNYIVEL GYORSABB?

1

FLoating point
Operation
Per
Second

FFT

$$2{,}7 \cdot 10^{10}\ s \approx 655\ yr$$

DFT

$$10^{18}\ s \approx 31\ Gyr$$

MENNYIVEL GYORSABB?

1

FLoating point
Operation
Per
Second

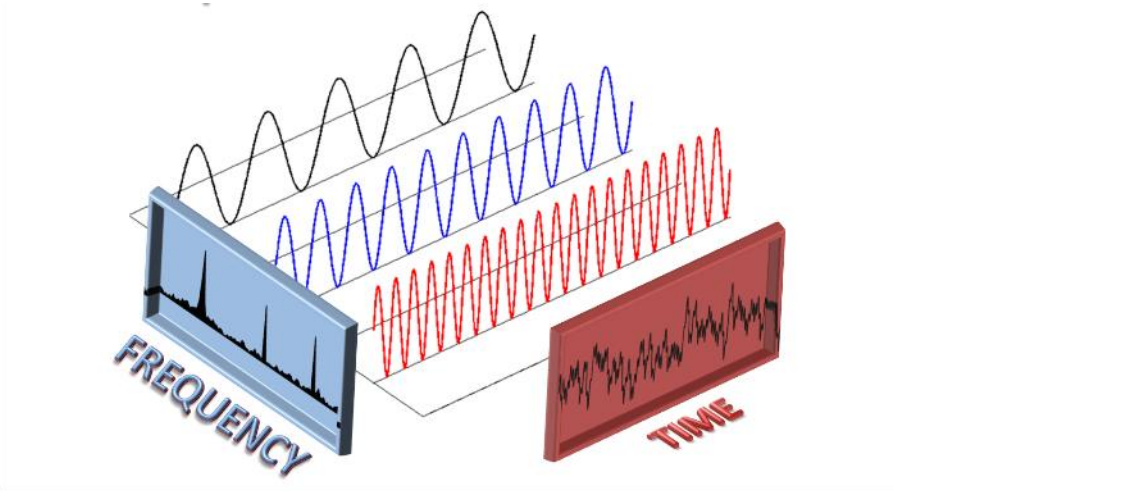
FFT

$2{,}7 \cdot 10^{10}\ s \approx 655\ yr$

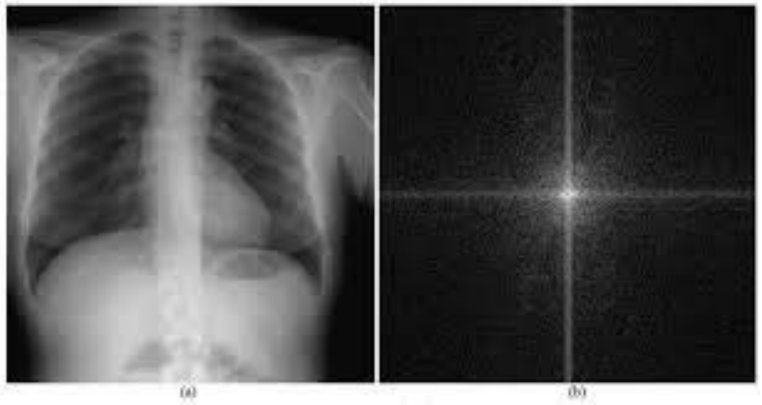
DFT

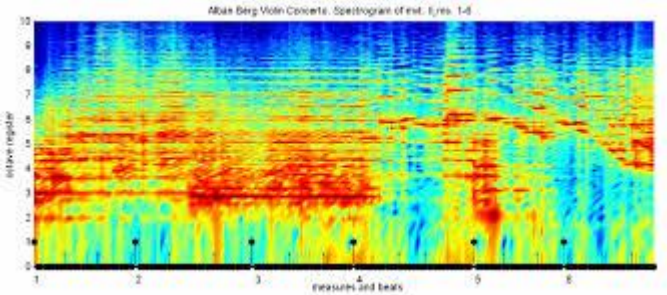
$10^{18}\ s \approx 31\ Gyr$

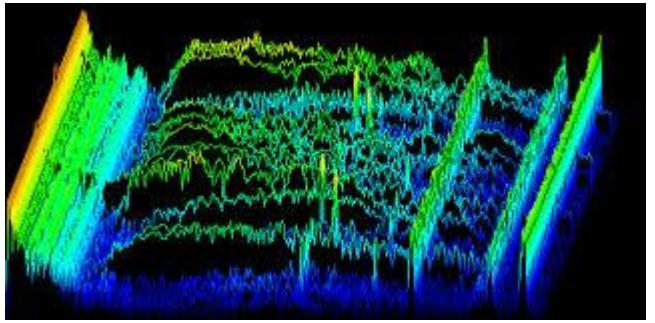
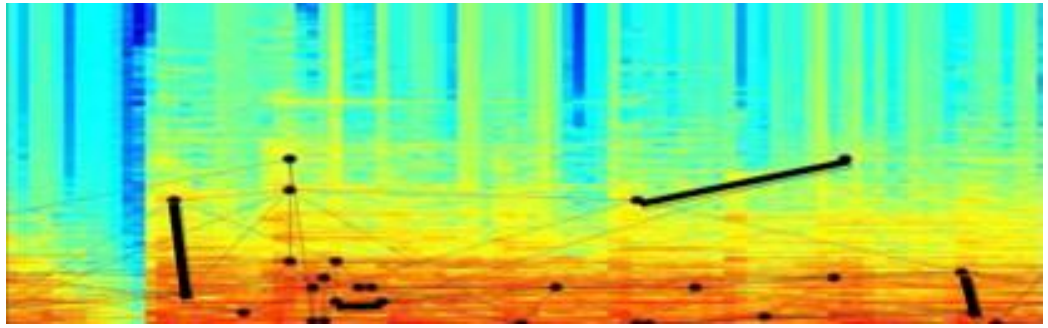
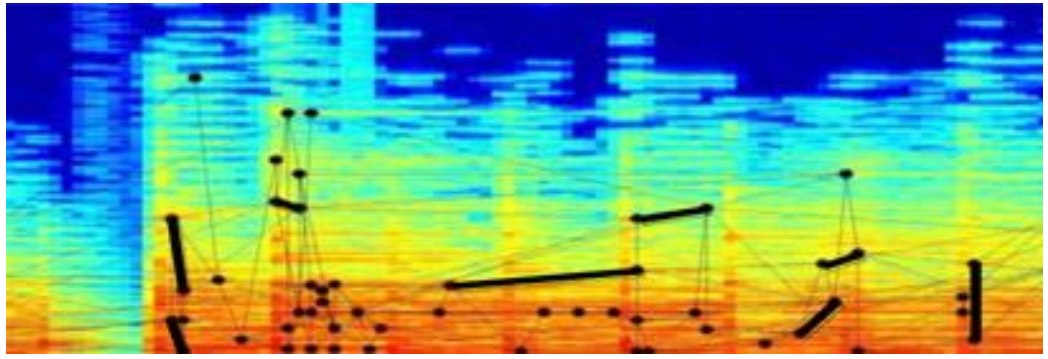
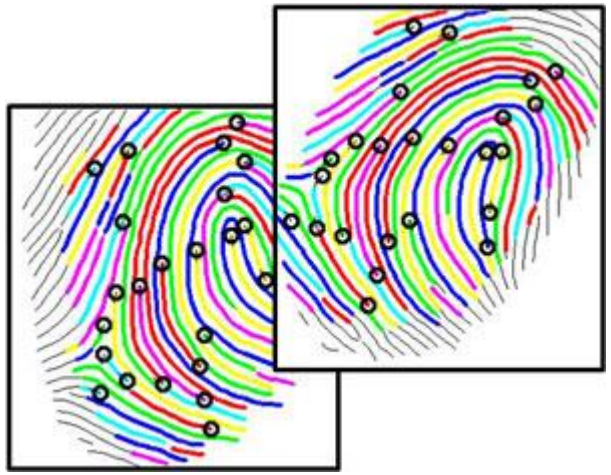
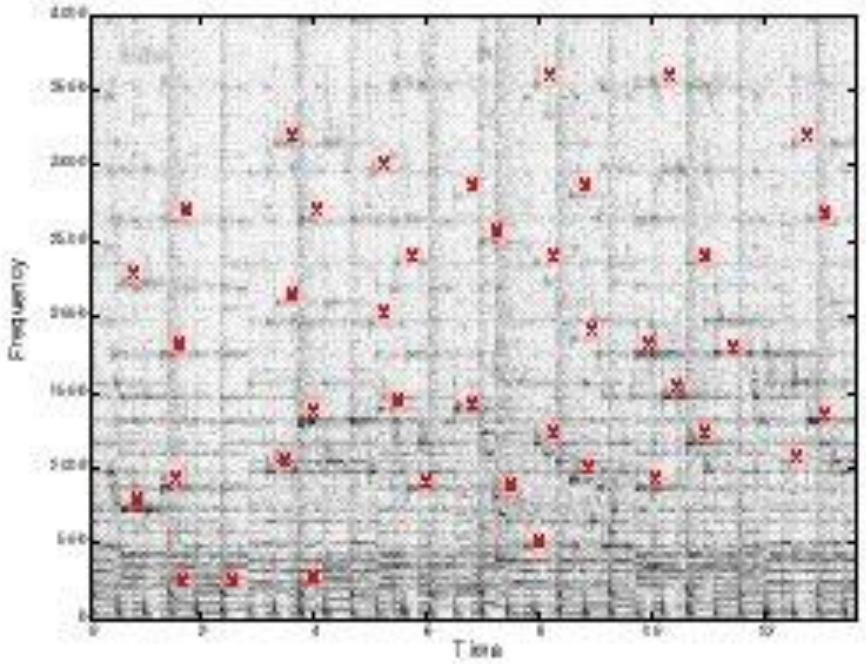
FFT MINDENRE, AMI JEL



FFT MINDENRE, AMI JEL







FFT MINDERE, AMI JEL