# Physical simulation based technique for determining continuous maps between meshes

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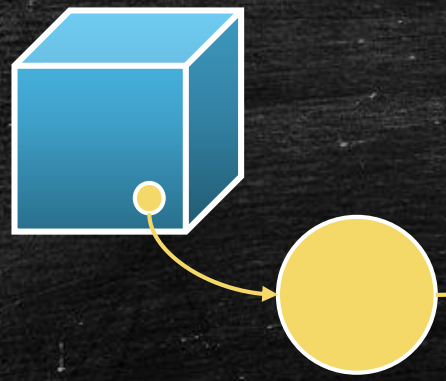
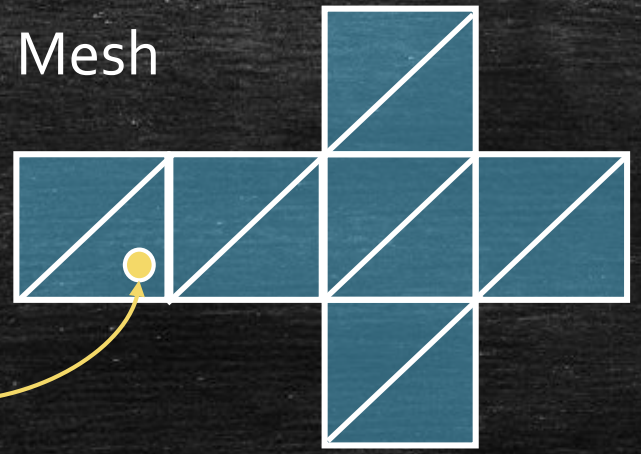
# Shape of the world

Object



Mesh



CG representation

```
v 0.0 0.0 1.0
v 0.0 0.0 0.0
v 1.0 0.0 0.0
v 1.0 0.0 1.0
v 0.0 1.0 1.0
v 0.0 1.0 0.0
v 1.0 1.0 0.0
v 1.0 1.0 1.0

f 2 1 5
f 2 5 6
f 6 7 3
f 6 3 2
f 7 8 4
f 7 4 3
f 1 4 8
f 1 8 5
f 1 2 3
f 1 3 4
f 8 7 6
f 8 6 5
```

**Surface**: orientable connected compact 2-manifold

- 2-manifold: … locally Euclidean of dimension 2.
- Compact: … every open cover has a finite cover.
- Connected: … does not exists a separation.
- Orientable: … has a surface presentation that has no twisted edge pairs.

# Shape of the world

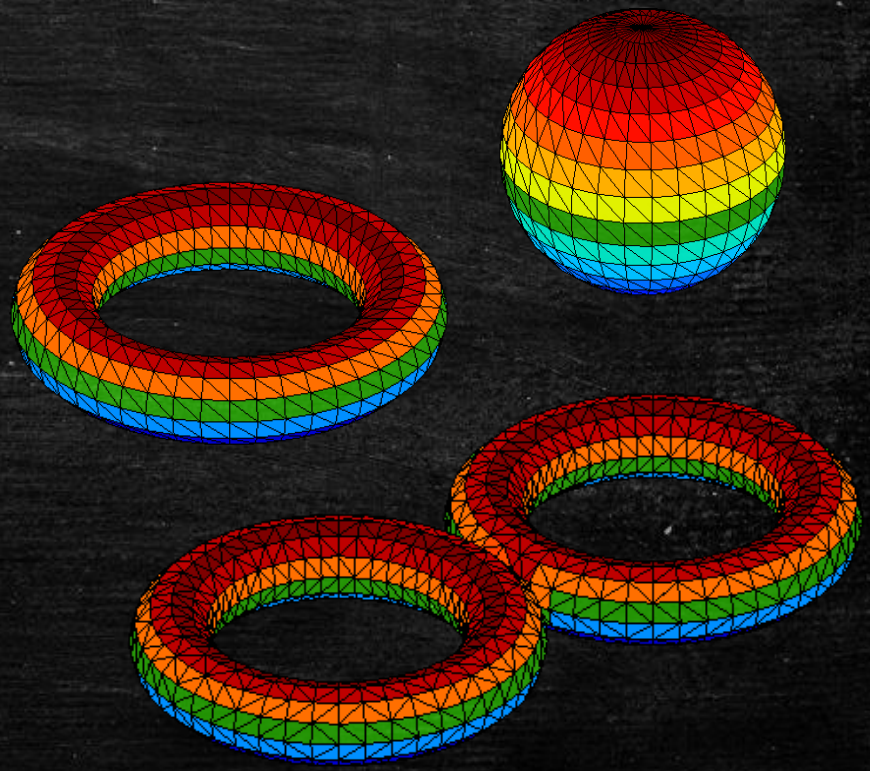
**Classification Theorem of Surfaces:** any connected compact orientable surface is homeomorphic to one of the following:

i) sphere
ii) connected sum of n tori

**The Euler characteristic**

$$\chi = V - E + F$$

$$\chi(X) \begin{cases} 2 & X \text{ is sphere} \\ 2 - 2n & X \text{ is n-tori} \end{cases}$$



# Homeomorphism & Homotopy

Homeomorphism is a map between topological spaces: $H : X \to Y$

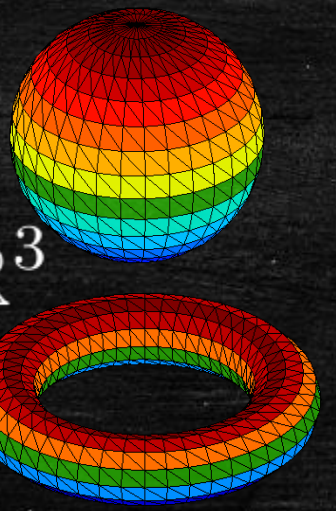
i) bijection
ii) continuous
iii) inverse continuous

Continuous deformation without cutting and gluing.

$$I \subset \mathbb{R}^2$$
$$f : I \to X$$
$$h(0, x) = f(x)$$

Homotopy

$$h : [0, 1] \times I \to \mathbb{R}^3$$

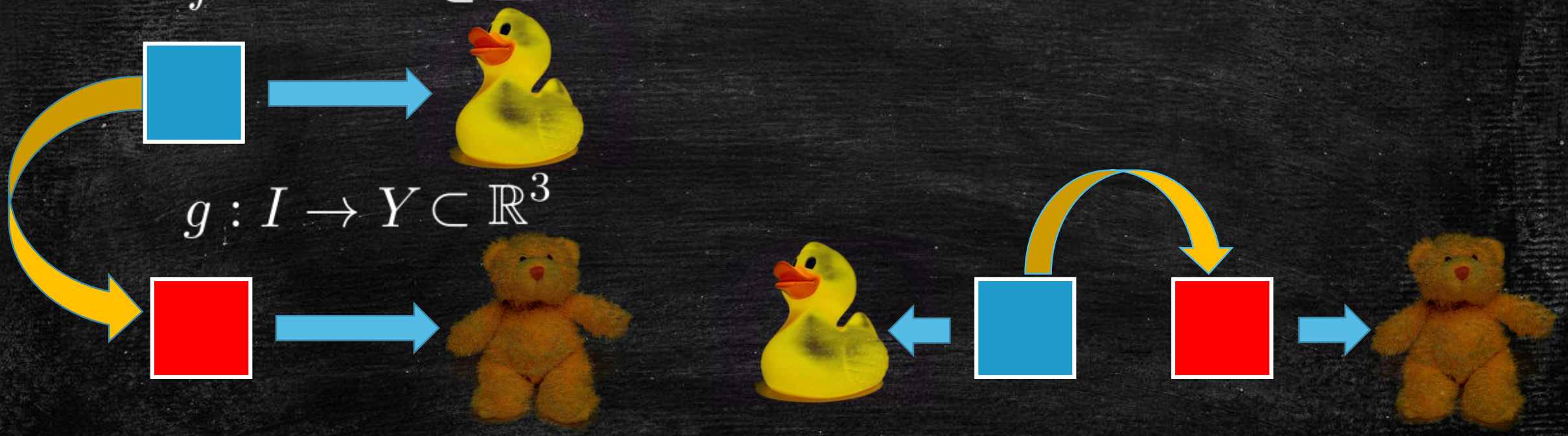


$$I \subset \mathbb{R}^2$$
$$g : I \to Y$$
$$h(1, x) = g(x)$$

# Transition between meshes

$$f : I \to X \subset \mathbb{R}^3$$

$$g : I \to Y \subset \mathbb{R}^3$$



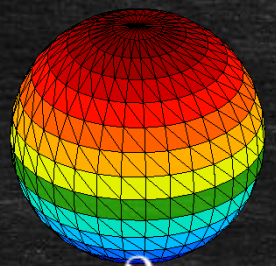
$$h : [0,1] \times I \to \mathbb{R}^3$$

$$h(t,x) = (1-t)f(x) + tg(x)$$

# Transition between meshes

$$H_X : X \to \mathbb{S}^2$$

$$f := H_X^{-1} \circ G$$

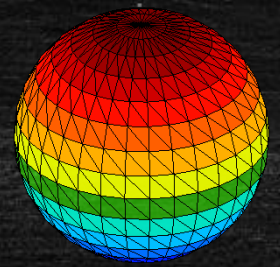


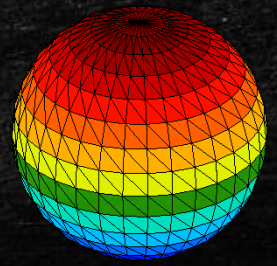
$$G(\theta, \phi) := \begin{pmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{pmatrix}$$

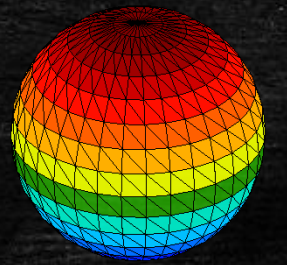
$$G : I \to \mathbb{S}^2$$

$$H_Y : Y \to \mathbb{S}^2$$

$$g := H_Y^{-1} \circ G$$

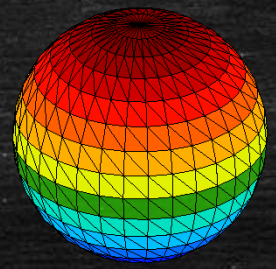


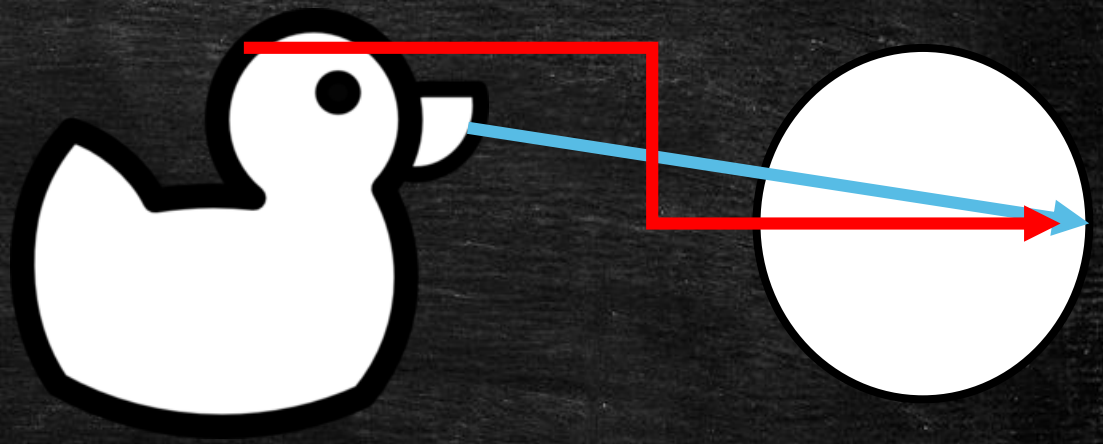




# What's wrong with projection?

?





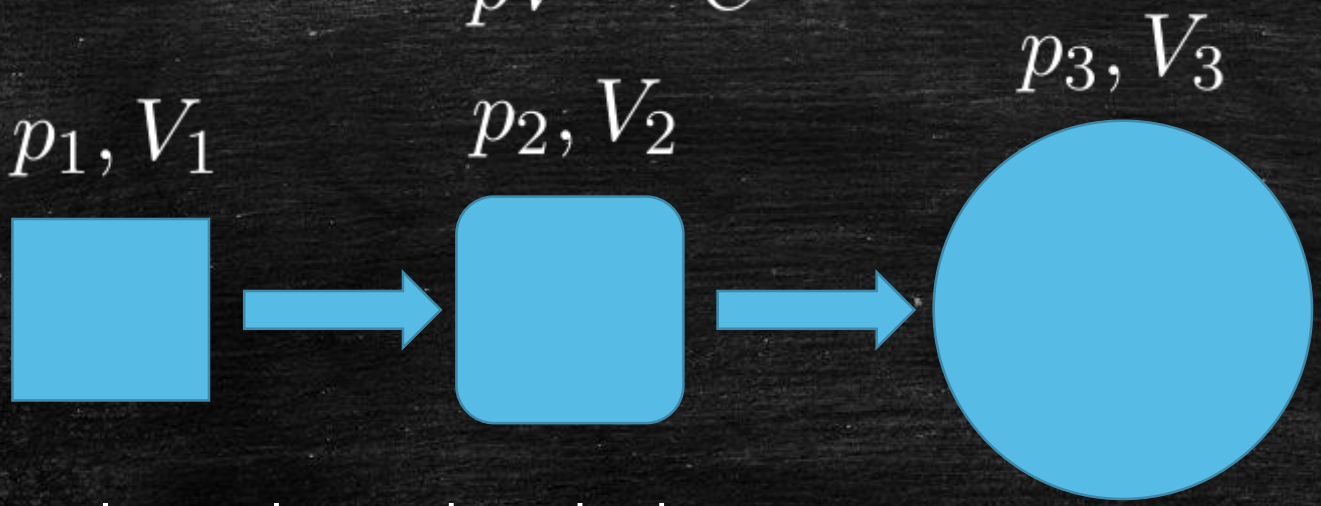
It is not a bijection!

What about the air balloon?

# Idea

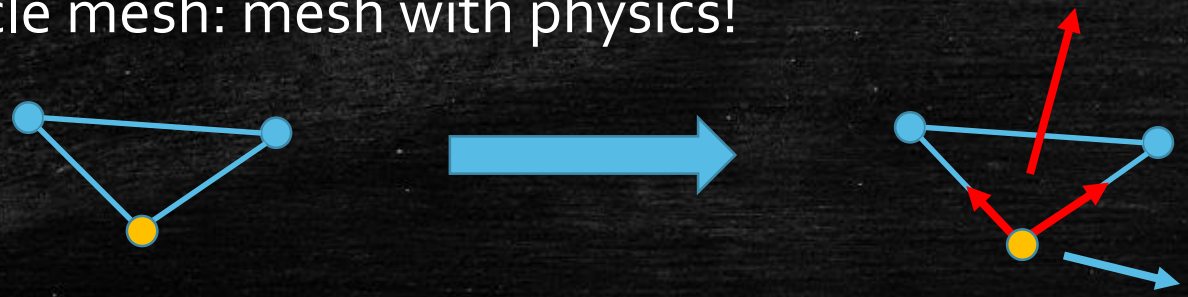
Isoperimetric problem → Under great pressure everything looks like a sphere!

Boyle-Mariotte law: $pV = C$

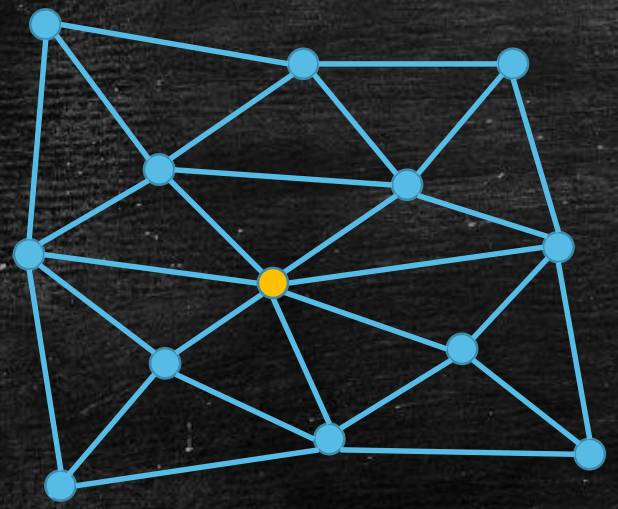


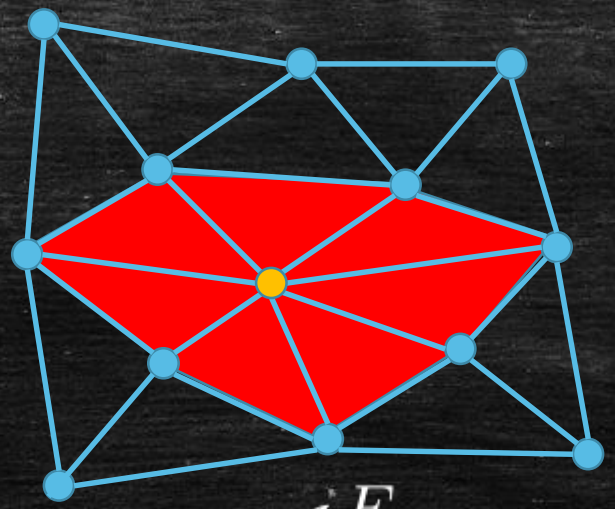

$$V_1 < V_2 < V_3$$
$$p_1 > p_2 > p_3$$

Particle mesh: mesh with physics!

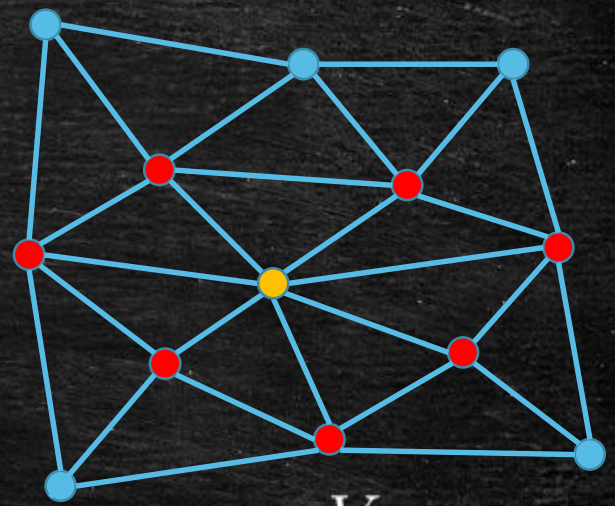


# Air balloon model



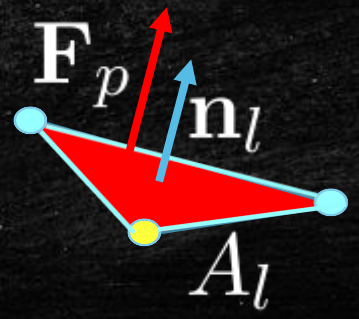


$\mathcal{A}_i^F$

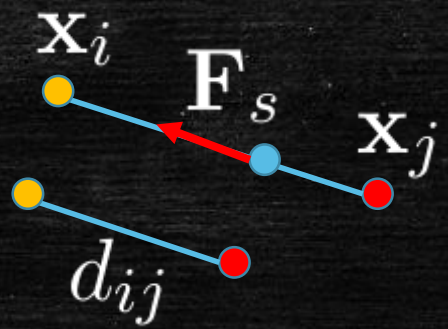


$\mathcal{A}_i^V$

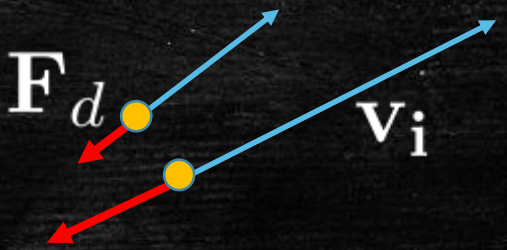
Pressure force




Spring force




Damping force



# Air balloon model

$$\mathbf{F}_p = pA\mathbf{n}$$

$$\mathbf{F}_s = -k(d_{ij} - \|\mathbf{x}_i - \mathbf{x}_j\|_2)\frac{\mathbf{x}_i - \mathbf{x}_j}{\|\mathbf{x}_i - \mathbf{x}_j\|_2}$$

$$\mathbf{F}_d = -c\mathbf{v}_i = -c\dot{\mathbf{x}}_i$$

$$\mathbf{F}_i = -k\sum_{j\in\mathcal{A}_i^V}(d_{ij} - \|\mathbf{x}_i - \mathbf{x}_j\|_2)\frac{\mathbf{x}_i - \mathbf{x}_j}{\|\mathbf{x}_i - \mathbf{x}_j\|_2} + p\sum_{l\in\mathcal{A}_i^F} A_l\mathbf{n}_l - c\mathbf{v}_i$$

$$\mathbf{F}_i = m_i\ddot{\mathbf{x}}_i = \ddot{\mathbf{x}}_i$$

$$\mathbf{F}_i = \dot{\mathbf{v}}_i$$
$$\mathbf{v}_i = \dot{\mathbf{x}}_i$$

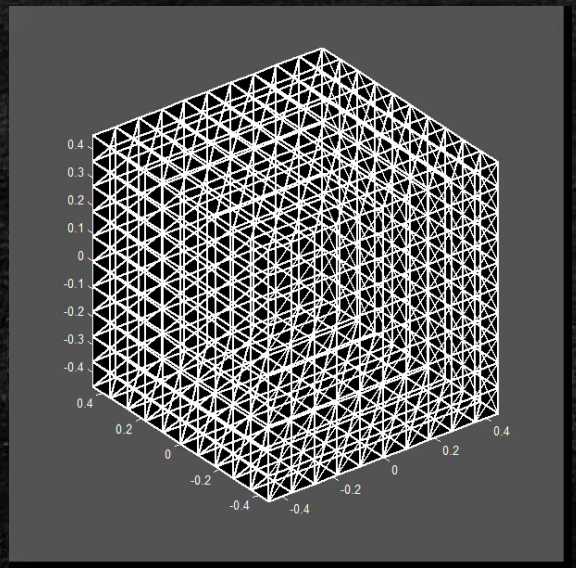
# Air balloon model

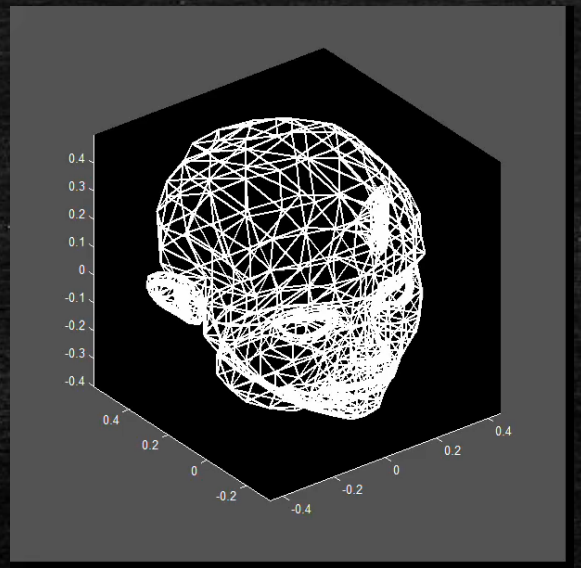
Differential equation system

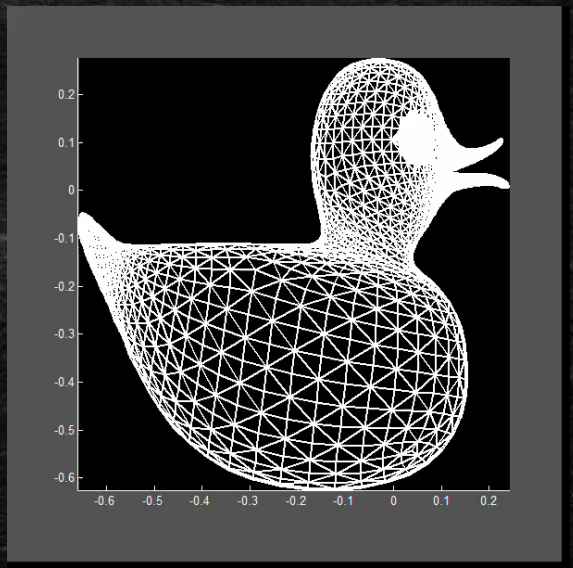
$$\mathbf{F}_i = \dot{\mathbf{v}}_i$$

$$\mathbf{v}_i = \dot{\mathbf{x}}_i$$

Solved by RK4 method



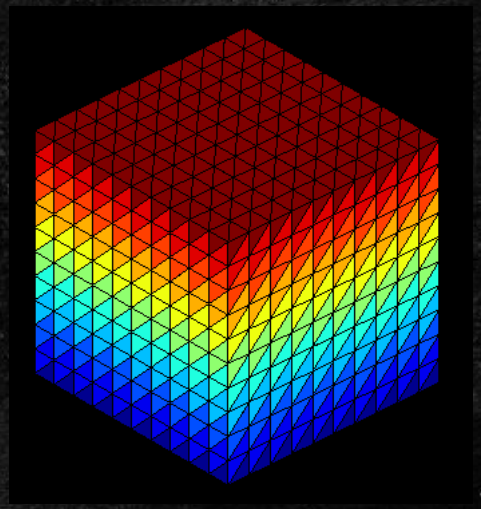
# Defining a homeomorphism

Vertex position sequence

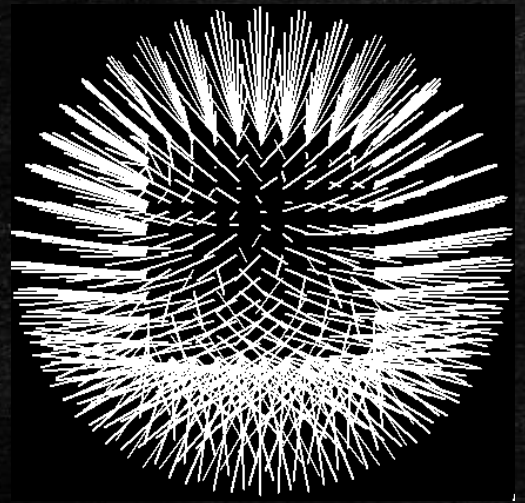
$$\mathbf{x}_i^{(0)}, \mathbf{x}_i^{(1)}, \ldots, \mathbf{x}_i^{(N)}$$

$$H_X(\mathbf{x}_i) = H_X(\mathbf{x}_i^{(0)}) = \frac{\mathbf{x}_i^{(N)}}{\|\mathbf{x}_i^{(N)}\|_2}$$

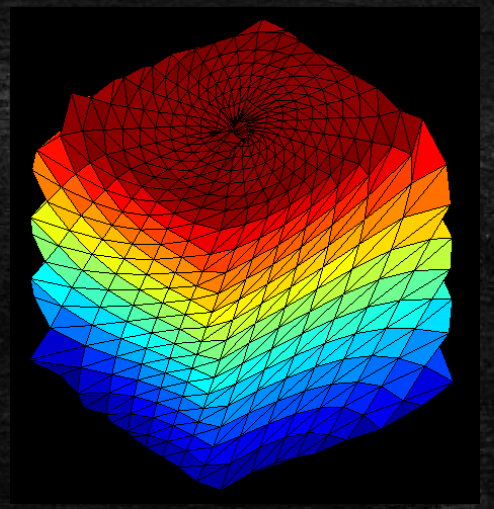
Input

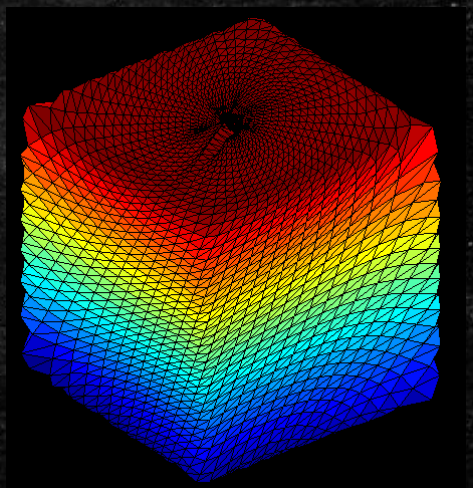


Displacements



Reconstruction





# Further plans

Avoid twisting (Optimization problem)

Smart stopping criteria

Generalization
- Torus
- n-tori

High level implementation
- Currently implemented in MATLAB

Parallel computation
- Slow iteration
- Easy parallelization

# Thank you for your attention!