ON CONSTRUCTING VOLUME BASED APPROXIMATION ALGORITHMS OF SPATIAL SUBSETS

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MOTIVATION

Computer graphics algorithms are often similar to each other.

- similar buliding blocks
- different representations of objects
- different goals
- not clear mathematical background

They can be unified by a general approximation schema depending on three functions.

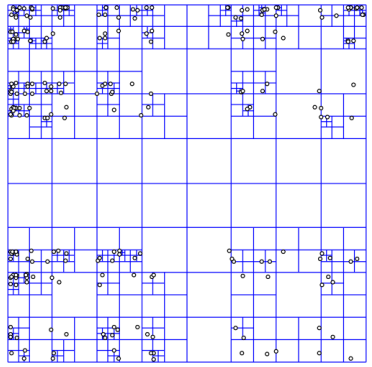
- well posed problems
- unified discussion
- constructing new algorithms

SPACE PARTITIONING

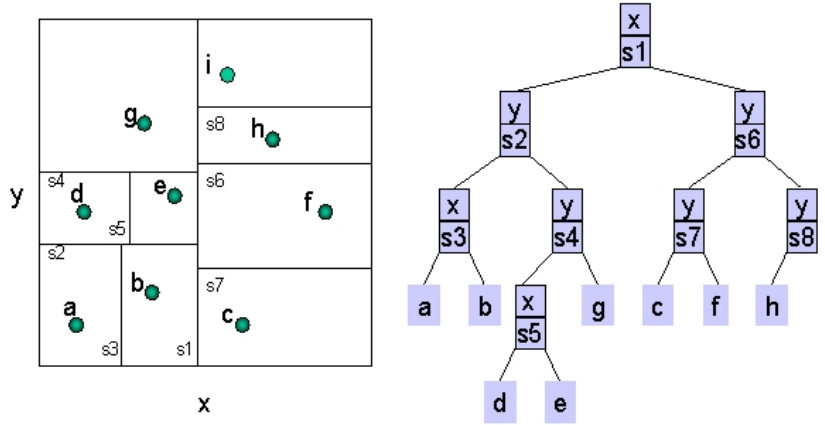
Hierarchical Data structures

- Quadtree
- Octree
- k-d tree
- BSP tree

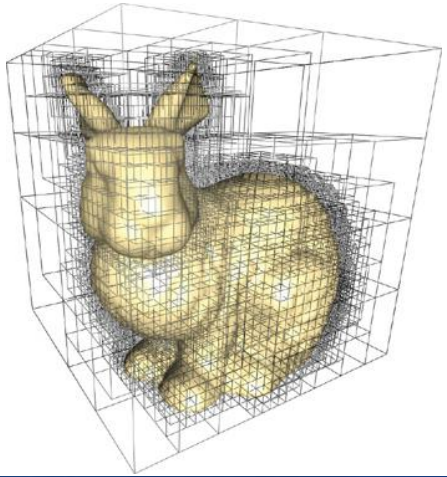
https://en.wikipedia.org/wiki/Quadtree



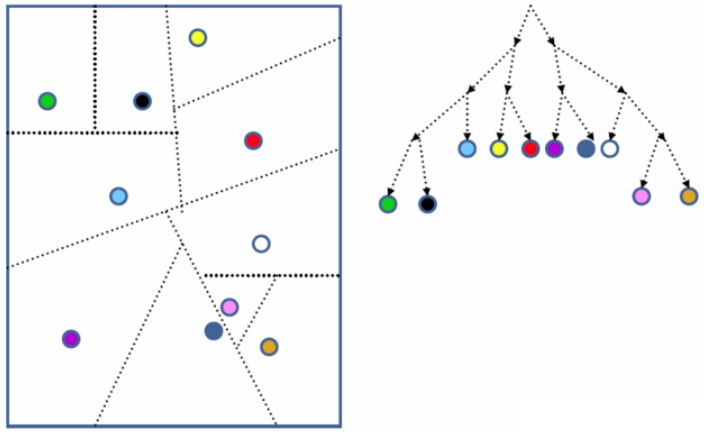
https://courses.cs.washington.edu/courses/csep521/99sp/lectures/lecture15/sld031.htm



http://http.developer.nvidia.com/GPUGems2/gpugems2_chapter37.html



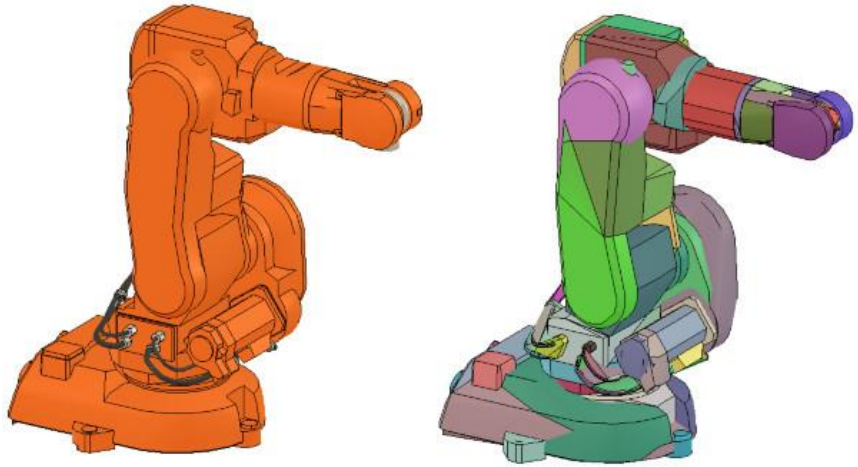
http://robotics.stackexchange.com/questions/2373/using-vision-for-monte-carlo-localization



CONVEX DECOMPOSITIONS

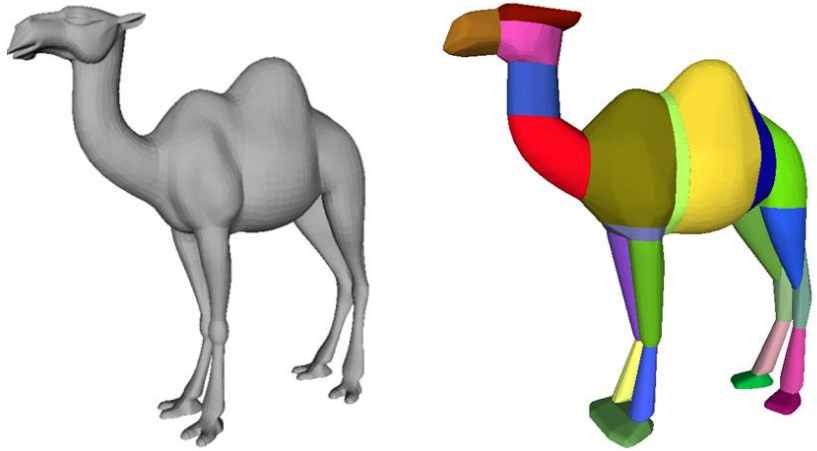
- Convex Decomposition
- Approximate Convex Decomposition

http://www.coppeliarobotics.com/helpFiles/en/convexDecomposition.htm



https://cs.gmu.edu/~jmlien/research/app-cd/appcd_2d.html

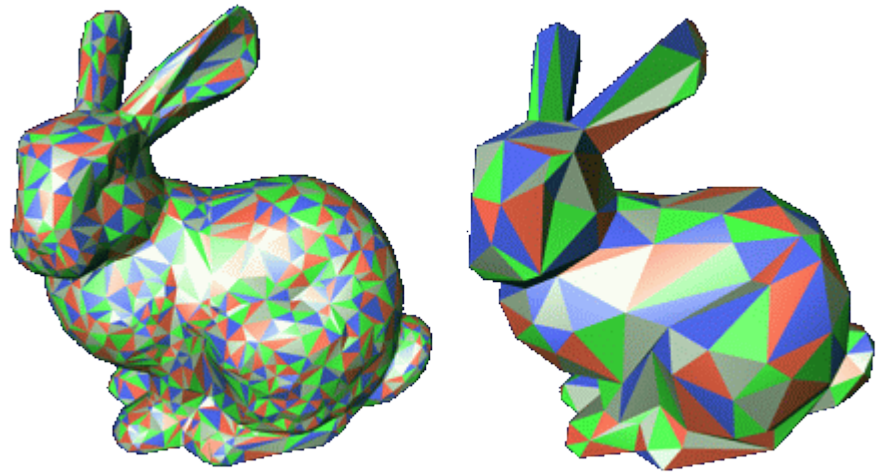
https://code.google.com/p/v-hacd/



MESH OPERATIONS

- Mesh Simplification
- Mesh Generation
- Mesh Repairing

http://vcg.isti.cnr.it/activities/surfacegrevis/multiresolution/zeta_html/intro.html



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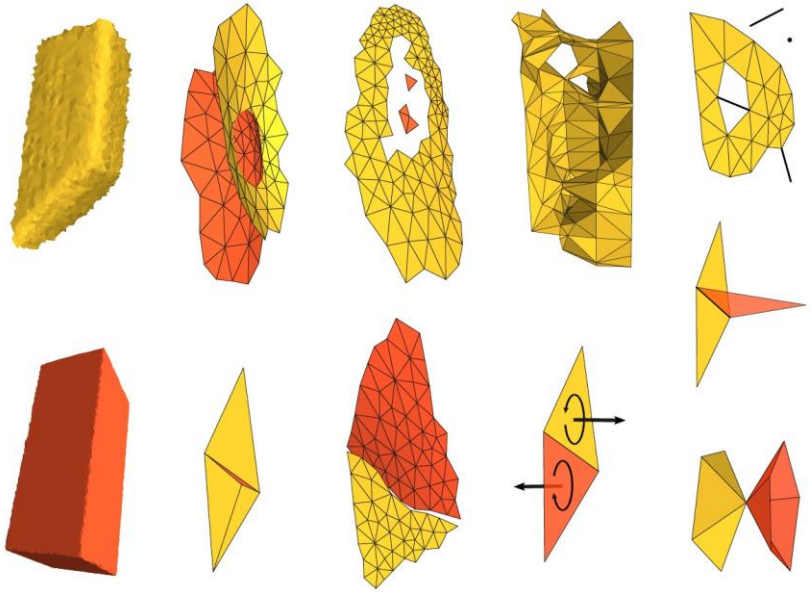
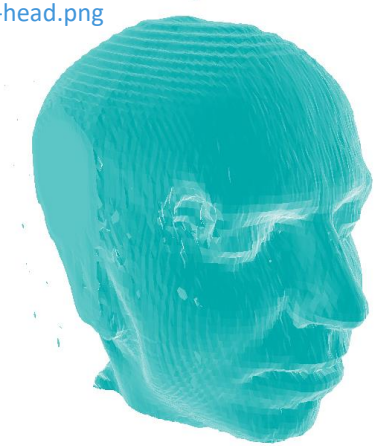


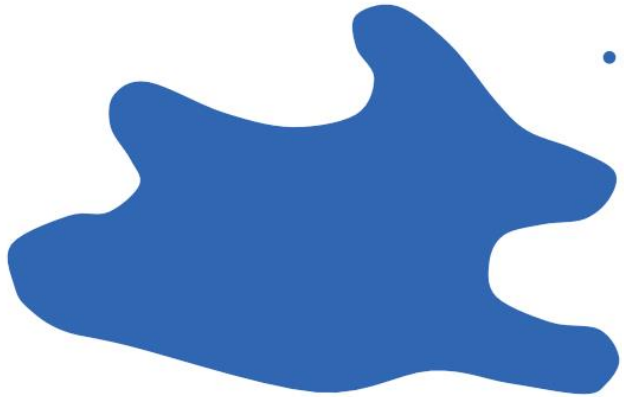
https://upload.wikimedia.org/wikipedia/commons/6/63/Marchingcubes-head.png



SPATIAL SUBSETS

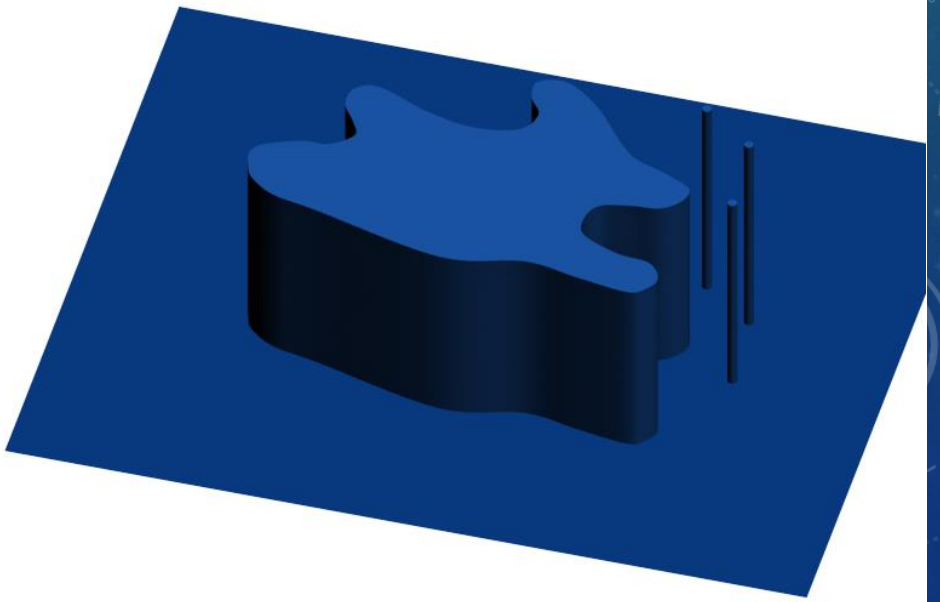
$$S \in \mathbb{B}^d \quad (d = 2, 3)$$

Functions vs Subsets



$$\chi_S : \mathbb{B}^d \to \{0, 1\}$$

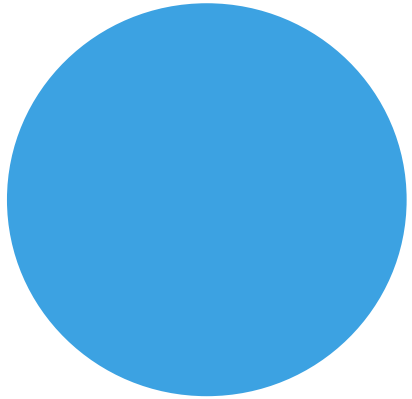
$$\chi_S(x) := \begin{cases} 1 & x \in S \\ 0 & x \notin S \end{cases}$$



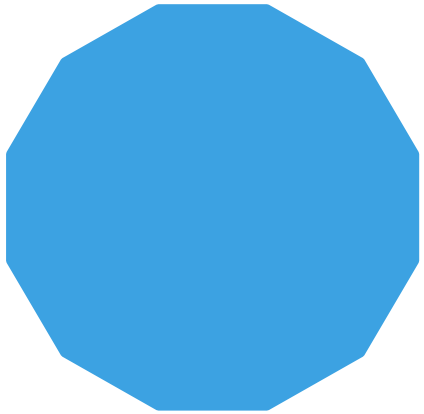
$$S = \chi_S^{-1}(1) = \{x \in \mathbb{B}^d \mid \chi_S(x) = 1\}$$

SOLIDS, SURFACES, MESHES

$S \in \mathbb{B}^d$



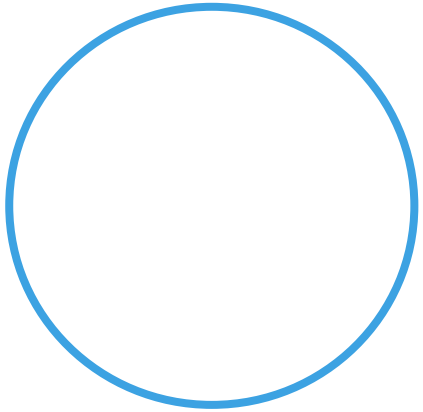
Triangulation



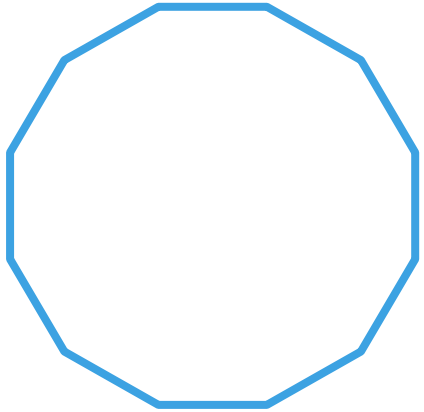
$\{x \in \mathbb{B}^d \mid f(x,y,z) \geq \alpha\}$

$\cup_{i=1}^{n} P_i$

$\partial S = \overline{S} \setminus \mathrm{int} S$



Triangulation



$\phi : [0,1]^{d-1} \rightarrow \mathbb{B}^d$

$\cup_{i=1}^{n} p_i$

$V \in \mathbb{R}^{m \times 3}$ $T \in \mathbb{R}^{n \times 3}$

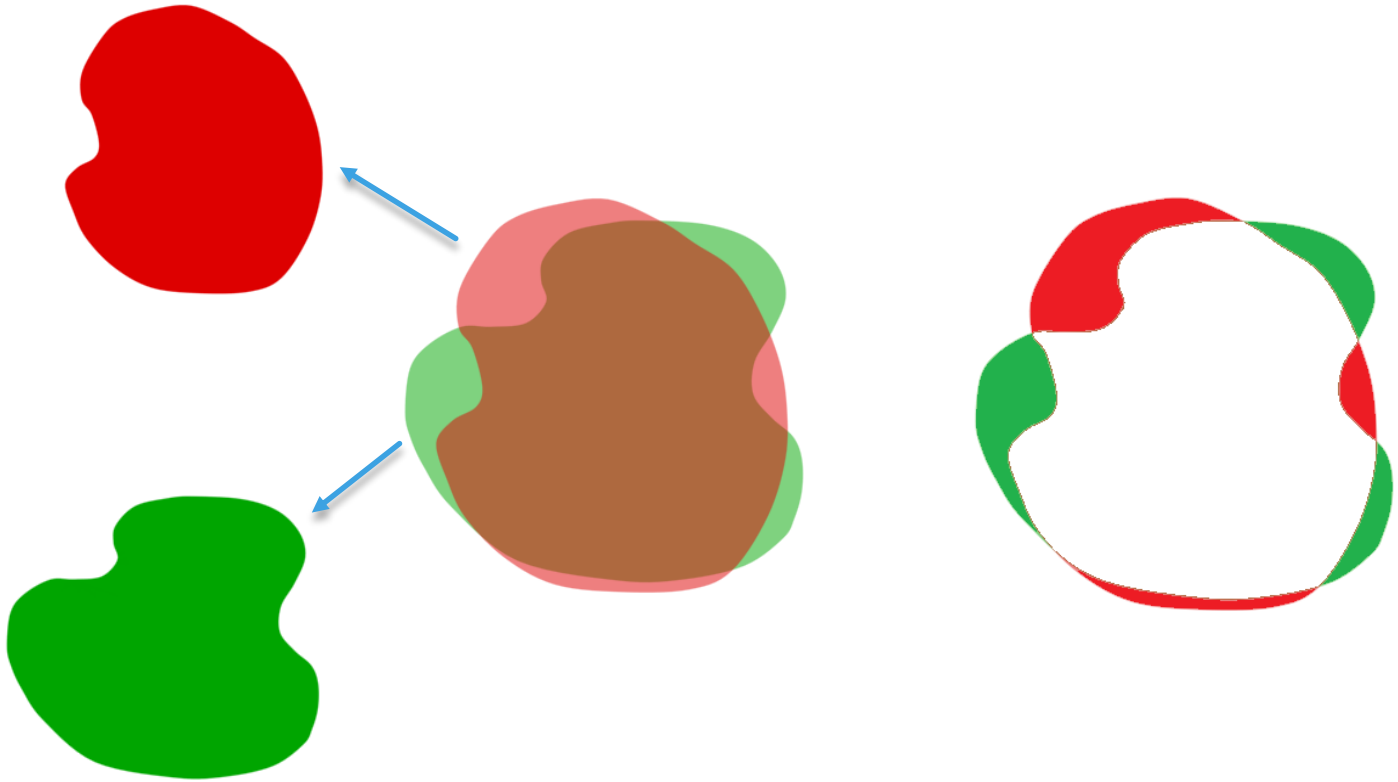
DISTANCE OF SUBSETS

$$A, B \in \mathbb{B}^d$$

$$\rho(A, B) := \mu(A \triangle B)$$

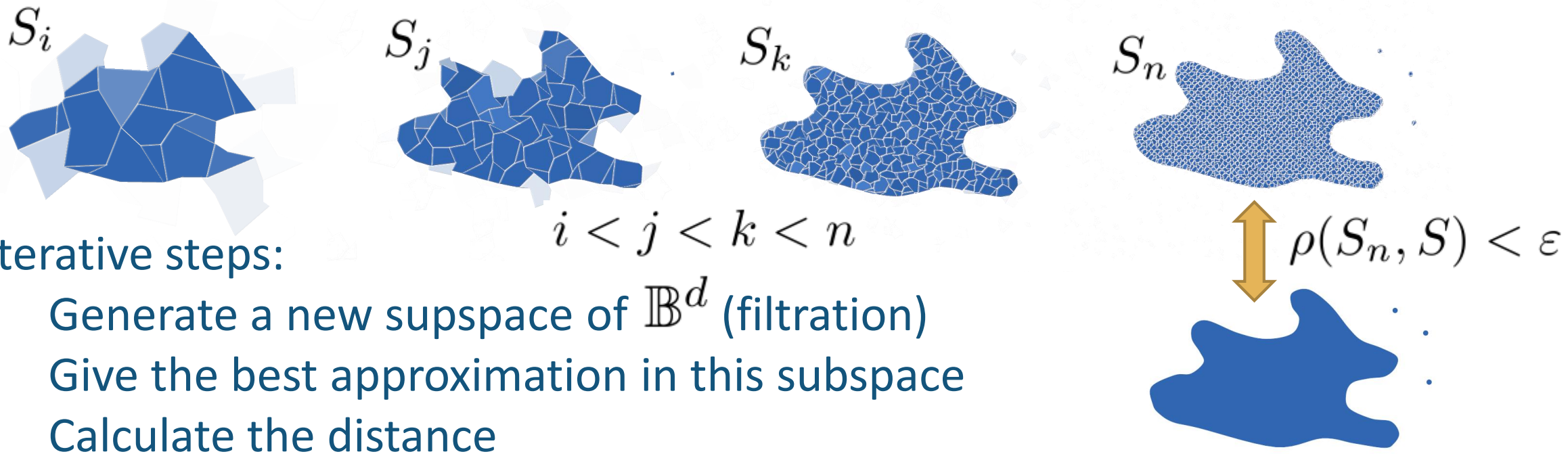
where μ is the

- counting measure
- d-dimensional Lebesgue-measure (area, volume)
- any measure



FINDING AN ALGORITHM

- give a representation
- simplify/modify representation
- give a decomposition



Iterative steps:

- Generate a new supspace of $\mathbb{B}^d$ (filtration)
- Give the best approximation in this subspace
- Calculate the distance

CONSTRUCTING A SUBSPACE

Atomic Decomposition:

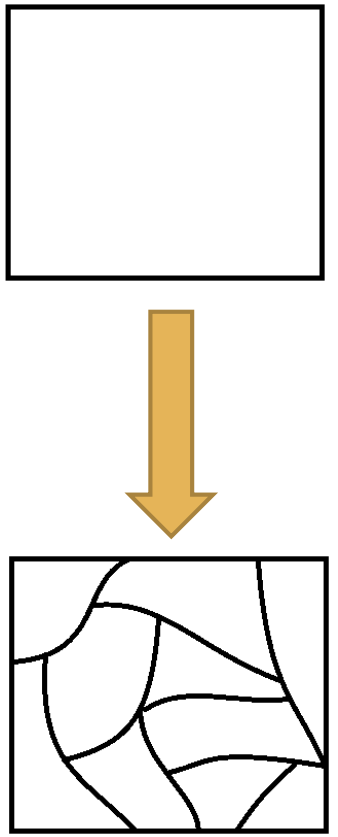
Pairwise disjoint sets covering the whole space

$$\mathcal{B}_n \subset 2^{\mathbb{B}^d}$$

$$\mathcal{B}_n := \sigma(B_i^{(n)} \subset \mathbb{B}^d \ : \ i = 0, 1, \ldots, n)$$

$$B_i^{(n)} \cap B_j^{(n)} = \emptyset \quad (i \neq j)$$

$$\cup_{i=0}^{n} B_i^{(n)} = \mathbb{B}^d.$$



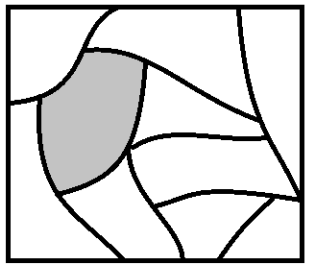
Atomic Decomposition is a closed subspace of a Hilbert-space.

BEST APPROXIMATION IN SUBSPACE

Riesz's projection theorem:
Best approximation in a closed subspace is the Fourier-series!

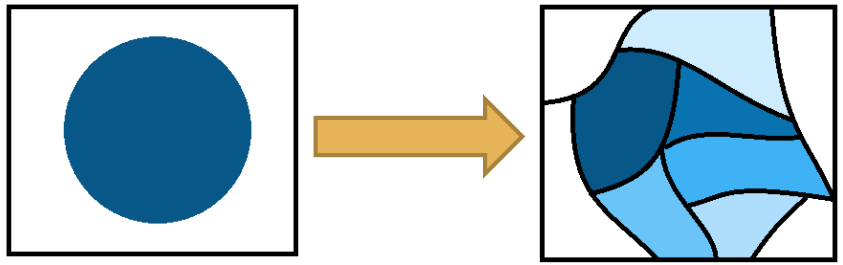
Orthonormed System

$$\phi_i^{(n)} := \frac{1}{\sqrt{\mu(B_i^{(n)})}} \chi_{B_i^{(n)}} \qquad (i = 0, 1, \ldots, n)$$



Fourier-series

$$\mathcal{F}^{\mathcal{B}_n} f := \sum_{i=0}^{n} \langle f, \phi_i^{(n)} \rangle \phi_i^{(n)}$$



BEST APPROXIMATION OF SUBSETS?

Riesz's projection theorem:

Best approximation in a closed subspace is the Fourier-series!

$$\mathcal{F}^{\mathcal{B}_n}\chi_S := \sum_{i=0}^{n} \langle \chi_S, \phi_i^{(n)} \rangle \phi_i^{(n)} = \sum_{i=0}^{n} b_i^{(n)} \chi_{B_i^{(n)}}$$

Fourier-coefficient:

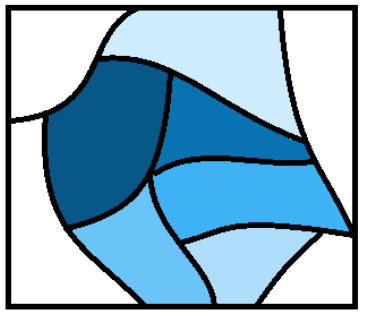
$$b_i^{(n)} := \frac{\mu(B_i^{(n)} \cap S)}{\mu(B_i^{(n)})} \in [0,1] \supset \{0,1\}$$

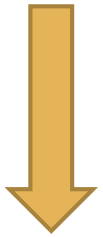
$\mathcal{F}^{\mathcal{B}_n}\chi_S$ is not a characteristic function! Then what is the subset?

DEFINITION OF THE APPROXIMATION

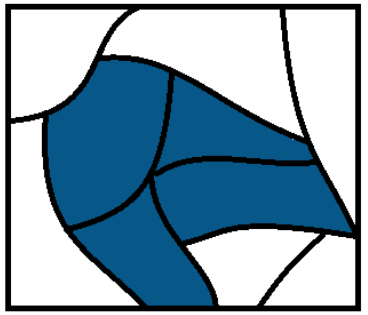
$$\mathcal{F}^{\mathcal{B}_n}\chi_S := \sum_{i=0}^{n}\langle\chi_S, \phi_i^{(n)}\rangle\phi_i^{(n)} = \sum_{i=0}^{n} b_i^{(n)}\chi_{B_i^{(n)}}$$

$$\mathcal{X}_\alpha f(x) := \begin{cases} 0 & f(x) \leq \alpha \\ 1 & f(x) > \alpha \end{cases}$$





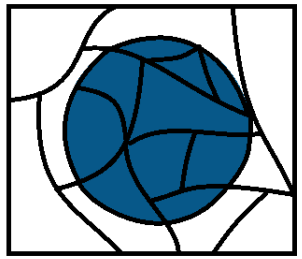
$$S_n := (\mathcal{X}_\alpha \mathcal{F}^{\mathcal{B}_n}\chi_S)^{-1}(1)$$



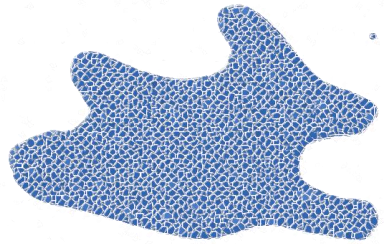
CONVERGENCE

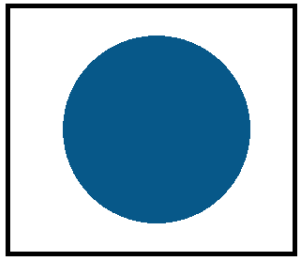
$$\Delta_n := \{i \in \{0, 1, \ldots, n\} \mid b_i^{(n)} \in (0, 1)\}$$

$$\operatorname{diam}(B) := \sup\{\|x - y\|_2 \mid x, y \in B\}$$

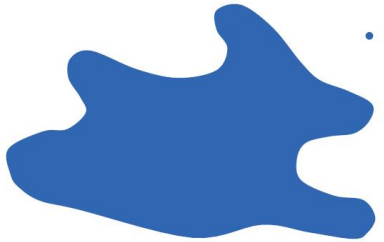


$$\lim_{n\to\infty} \max_{k\in\Delta_n} \operatorname{diam}(B_k^{(n)}) = 0$$





$$\lim_{n\to\infty} \rho(S_n, S) = 0$$



OUTLINE OF THE ALGORITHM

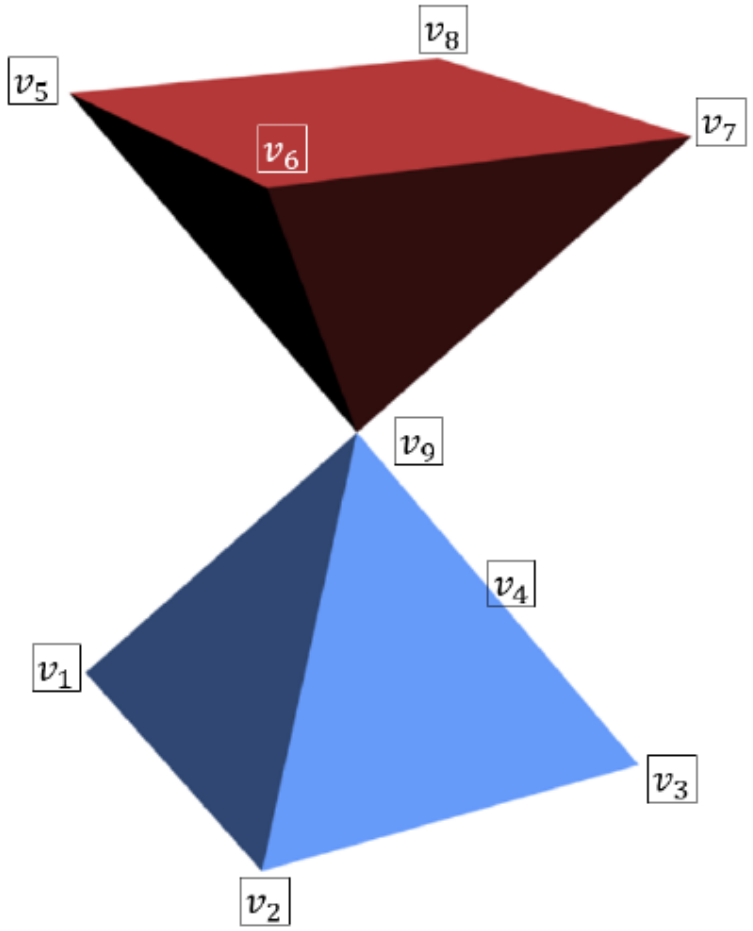
VolumeBasedApproximation$(S, B_0^{(0)}, \varepsilon, \mathcal{C}, \mathcal{D})$

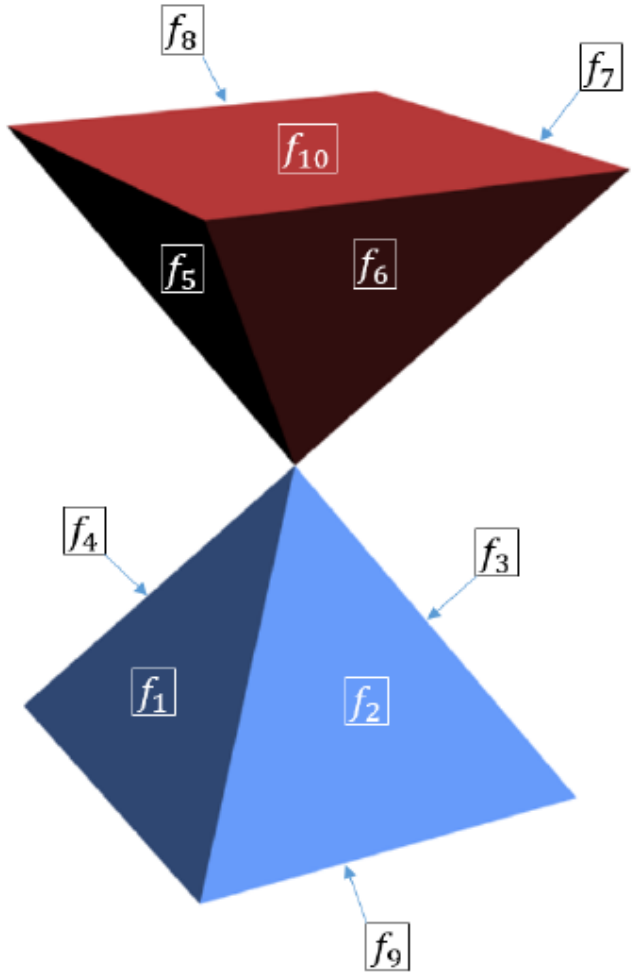
1. $n := 0$
2. $\mathcal{B}_n := \sigma(B_0^{(n)}, \ldots, B_n^{(n)})$
3. $k := \mathcal{C}(\mathcal{B}_n)$
4. $S_n := \{\mathcal{X}_\alpha \mathcal{F}^{\mathcal{B}_n} \chi_S = 1\}$
5. **if** $\rho(S_n, S) \geq \varepsilon$ **then**
6. $\quad \mathcal{B}_{n+1} := \sigma(\ldots, B_{k-1}^{(n)}, \mathcal{D}(B_k^{(n)}), B_{k+1}^{(n)}, \ldots)$
7. $\quad n := n + 1$
8. $\quad$ **goto** 3.
9. $\quad$ **else stop**

$$\forall n \in \mathbb{N} \quad \mathcal{C}(\mathcal{B}_n) \in \Delta_n$$

$$\forall B \in \mathbb{B}^d \quad \mathcal{D}(B) = (B', B''), \quad B' \cap B'' = \emptyset, \; B' \cup B'' = B$$

DATA STRUCTURE BEHIND

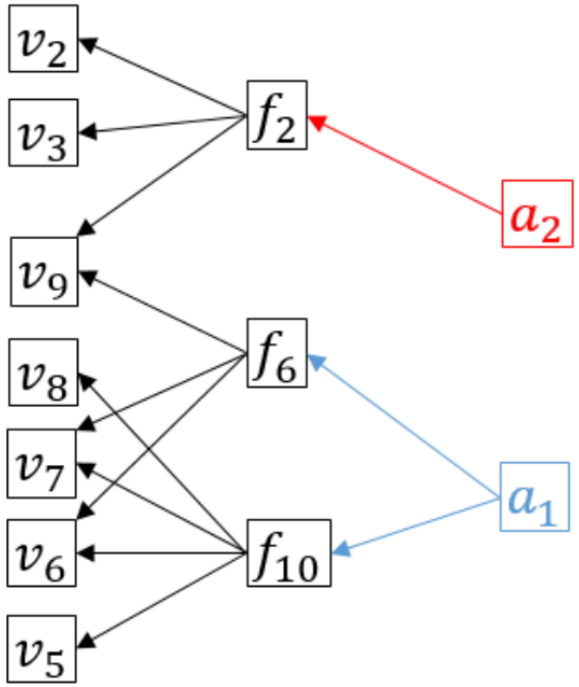




V: coordinates

F: vertex indexes

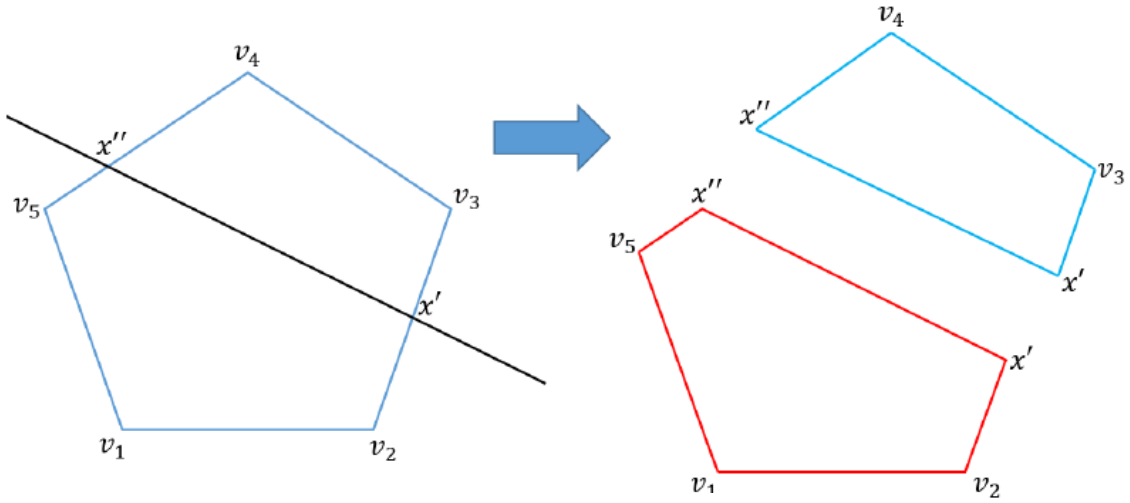
A: face indexes



DIVIDING OPERATION

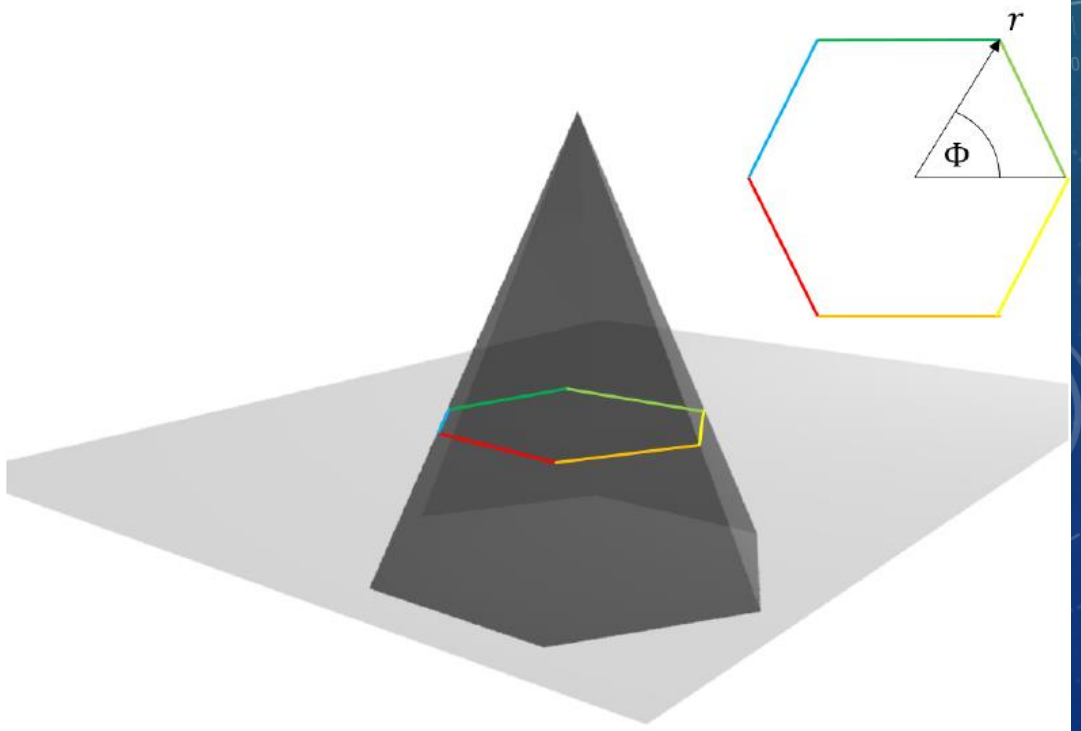
$$P_{n,a} := \{x \in \mathbb{R}^3 \mid \langle x - a, n \rangle = 0\}$$

Dividing by a plane conserves the convexity!



$$x' := v_i + \frac{\langle n, a - v_i \rangle}{\langle n, v_{i+1} - v_i \rangle}(v_{i+1} - v_i)$$

$$x'' := v_j + \frac{\langle n, a - v_j \rangle}{\langle n, v_{j+1} - v_i \rangle}(v_{j+1} - v_j)$$



SETTING UP INPUT PARAMETERS

$B_0^{(0)}$ be the bounding box of input set S

$\mathcal{C}$

random atom
maximum volume atom
maximum diameter atom
atom containing most vertices
…

$\mathcal{D}$

random plane at center
plane from random triangle
least coplanar plane
particularly best fitting plane
…

GETTING RESULTS

$$A_n := \{i \in [0..n] \ : \ b_i^{(n)} \geq \alpha\}$$

$$S \approx S_n = \bigcup_{i \in A_n} B_i^{(n)}$$

$$B_i^{(n)} \sim a_i$$

$$\partial B_i^{(n)} \sim \bigcup_{k \in a_i} f_k$$

$$(V, F, A)$$

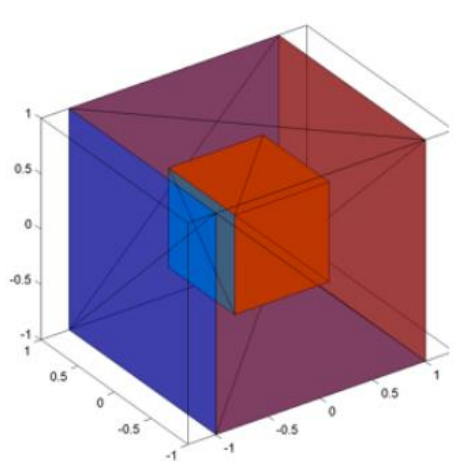
$$(V_1, T_1) \qquad (V, T)$$

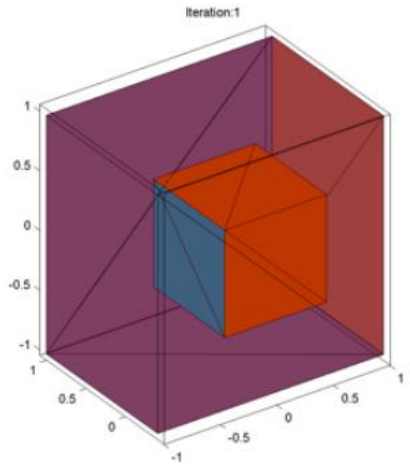
$$(V_2, T_2)$$

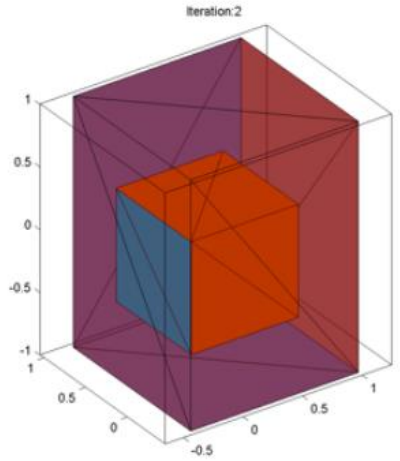
...

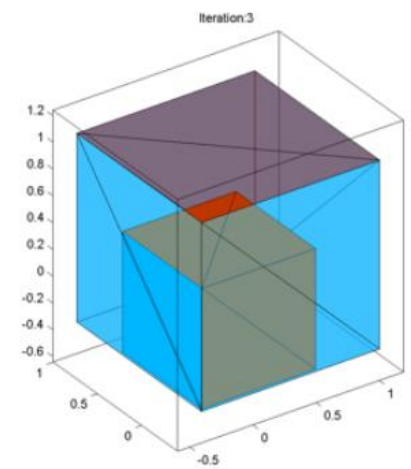
- Single mesh
- Multiple meshes
- Solid
- Hierarchical data structure

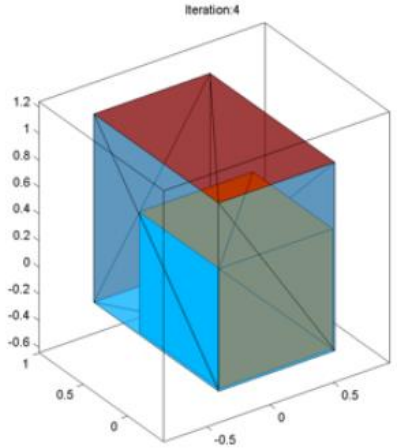
EXPERIMENTS – TRIVIAL

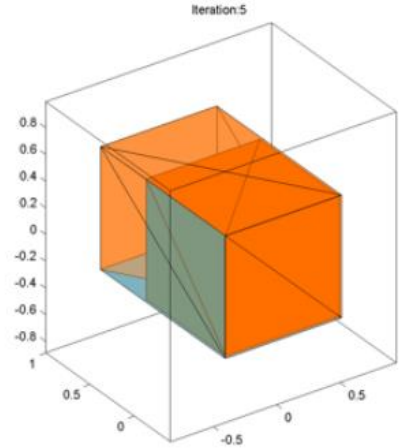


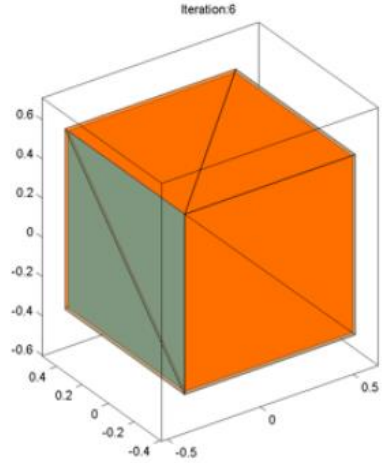


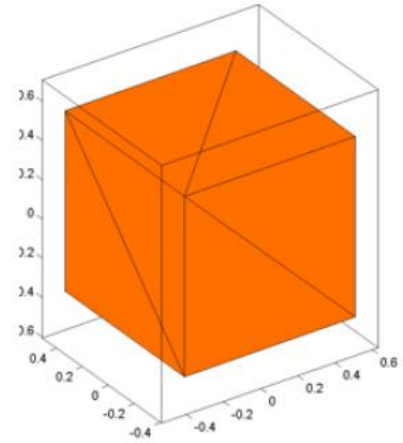




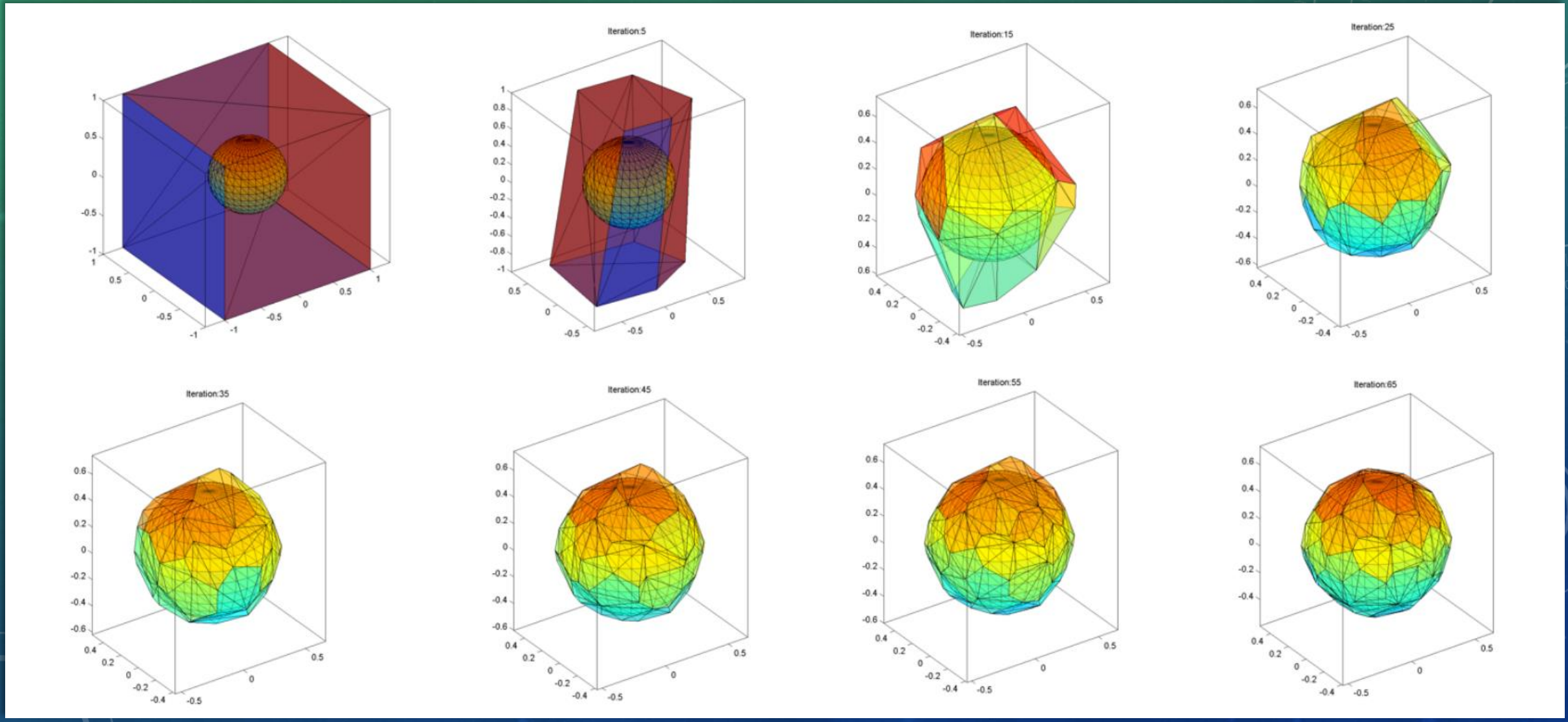








EXPERIMENTS – MESH SIMPLIFICATION



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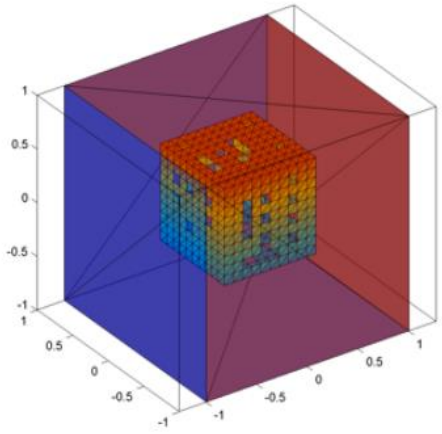
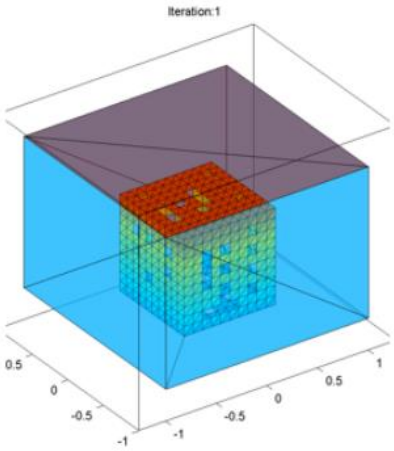
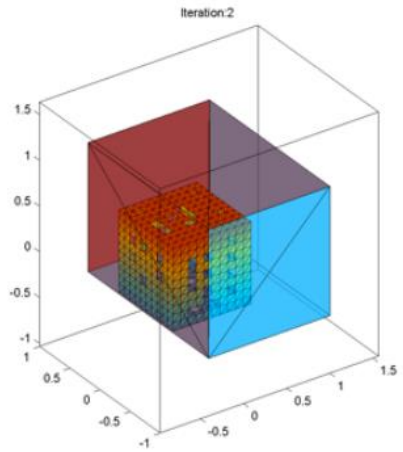
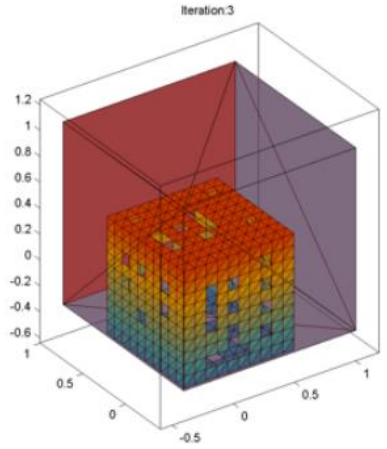
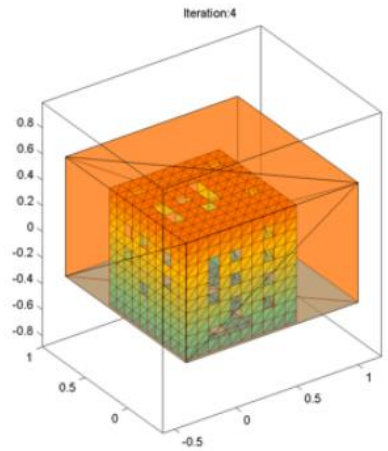
EXPERIMENTS – MESH REPAIRING

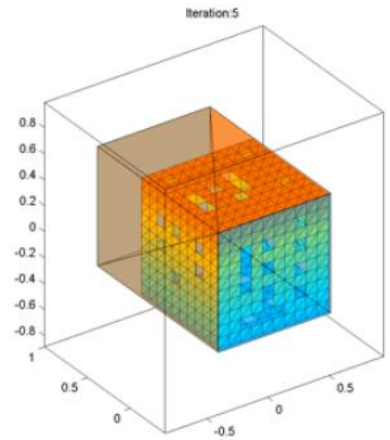


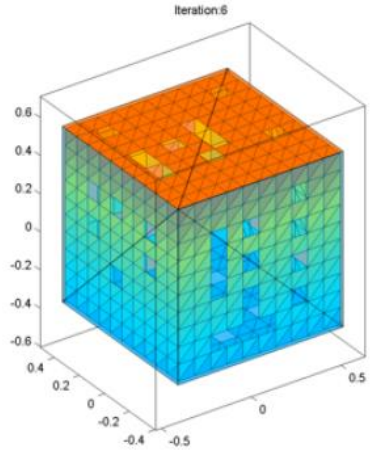


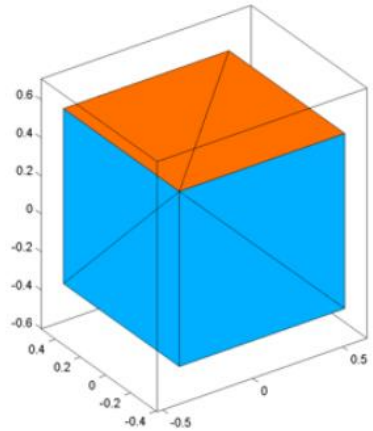












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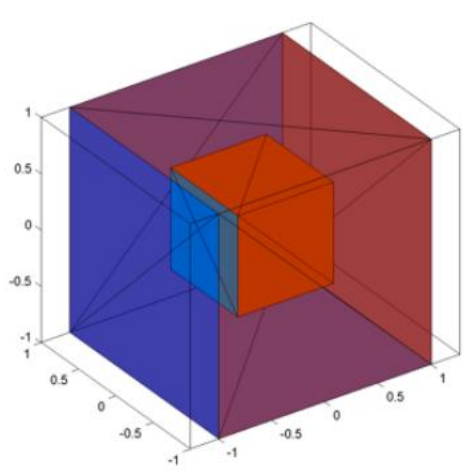
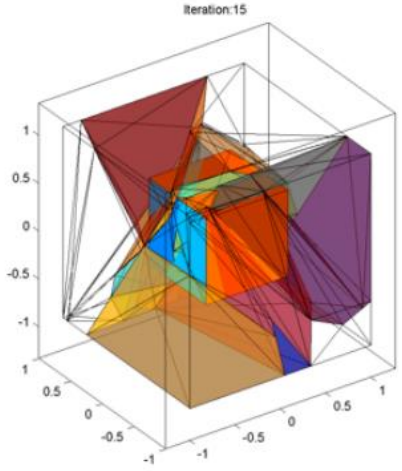
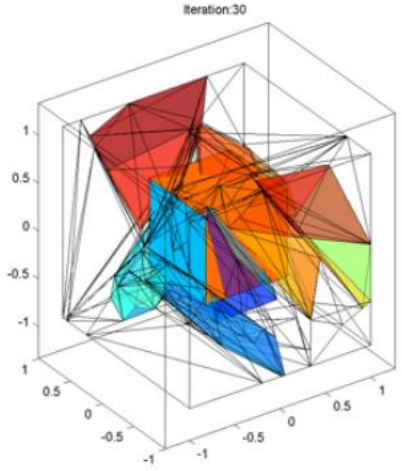
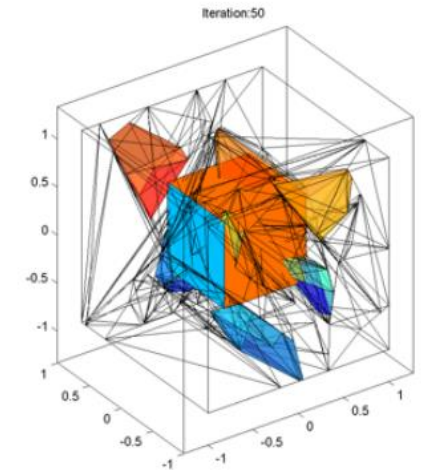
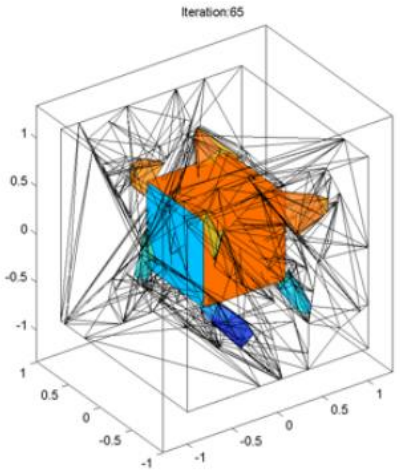
EXPERIMENTS – SPACE PARTITIONING

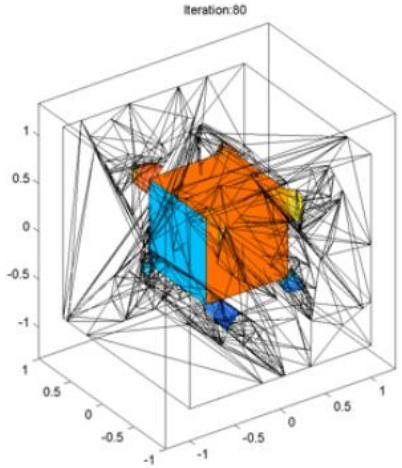


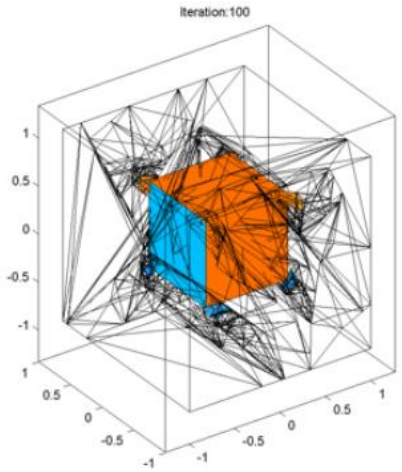


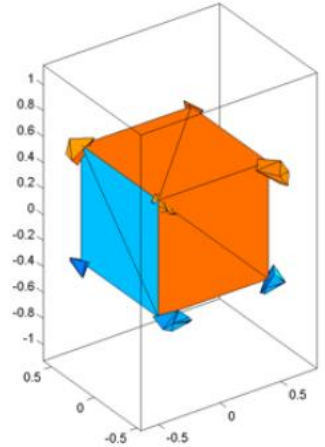












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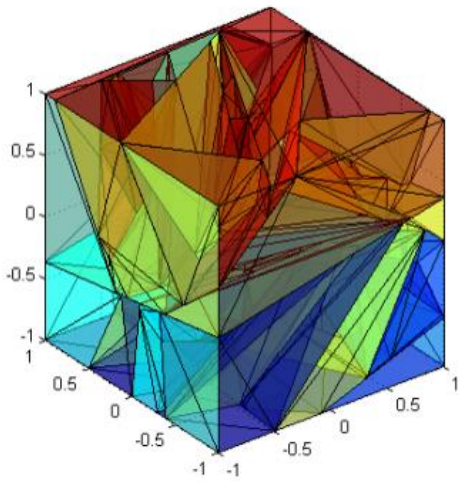
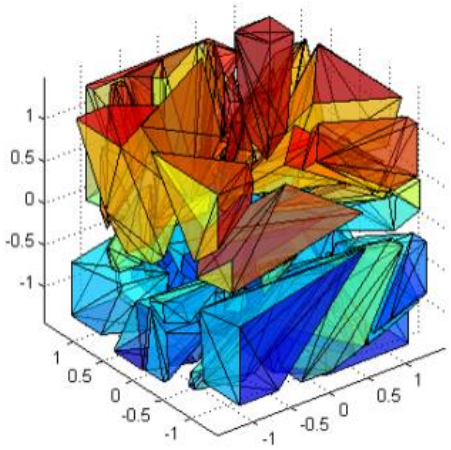
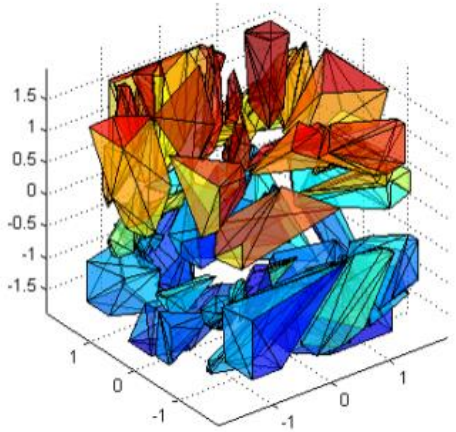
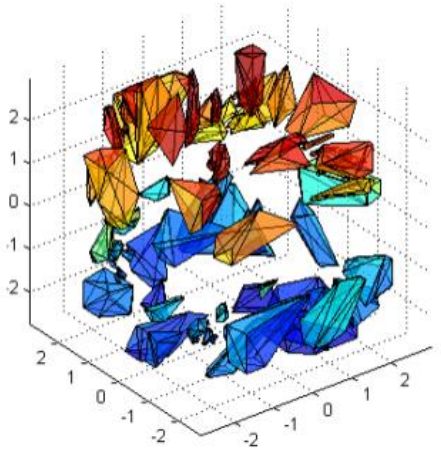
EXPERIMENTS – FRACTURE SIMULATION









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THANK YOU FOR YOUR ATTENTION!

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